A dialogical, multi-agent account of the normativity of logic

Catarina Dutilh Novaes

Faculty of Philosophy, University of Groningen

c.dutilh.novaes@rug.nl

Abstract

The paper argues that much of the difficulty with making progress on the issue of the normativity of logic for thought, as discussed in the literature, stems from a misapprehension of what logic is normative for. The claim is that, rather than mono-agent mental processes, (deductive) logic in fact comprises norms for quite specific situations of multi-agent dialogical interactions, in particular special forms of debates. This reconceptualization is inspired by historical developments in logic and mathematics (briefly surveyed in the paper), in particular the pervasiveness of such dialogical conceptions in the early days of logic in ancient Greece. The multi-agent, dialogical perspective then allows for the formulation of compelling 'bridge principles' between the relation of logical consequence and dialogical normative principles, something that is notoriously difficult to achieve in a mono-agent setting pertaining exclusively to thought and belief. The upshot is also that the truth-preserving rules of logic generally do not have a primary normative bearing on mono-agent mental processes, and in this sense the paper sides with Harman's (1986, 2009) critique of the idea that logic has a normative import for thought and belief.

1. Introduction

According to a familiar slogan (found for example in dictionaries and encyclopedias), logic is the science (or art) of correct reasoning: it studies the principles of acceptability for reasoning. The emphasis is on correct reasoning: on this view, logic has prescriptive – normative – rather than descriptive import for reasoning. One of the appealing features of this conception is that it seems to provide the means for a characterization of logic as distinct from other fields of inquiry such as psychology, mathematics or linguistics.

Yet, the precise nature of the (putative) normative import of logic is still debated among philosophers. In particular, what exactly are the phenomena falling under the normative grip of logic? 'Reasoning' is an ambiguous term, as it can pertain both to multi-agent, linguistic, social situations of argumentation ("You can't reason with her!"), and to inner mental processes of thinking (Mackenzie 1989, 100). However, in recent debates, the focus has been almost exclusively on the normativity of logic specifically for *thought*, i.e. mono-agent mental processes regulating beliefs. But even if one is prepared to grant logic a normative status, there is still the question of why *thought* specifically is what falls under its normative grip. Depending on one's answer to the question of what logic is normative *for*, different accounts of the nature of this normativity will be required. Questions such as 'where do the laws of logic come from?' are bound to be more adequately treated once the presumed normative remit of logic is clearly delineated.²

In this paper, I argue that much of the difficulty with making progress on these issues stems from a misapprehension of what logic is normative *for*. The claim is that, rather than for mono-agent mental processes, (deductive) logic in fact comprises norms for specific

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¹ To be clear, there is also much that we may want to refer to as 'thinking' that does not fall under the category of reasoning.

² For reasons of space, I leave aside non-normative, realist conceptions of logic such as Bolzano's. The ² For reasons of space, I leave aside non-normative, realist conceptions of logic such as Bolzano's. The main thesis of this paper may thus be formulated as a conditional claim: *if* logic has normative import at all, then the present proposal is a compelling way to make sense of this idea. (I do not claim to establish the antecedent of the conditional.)

situations of *dialogical interaction*, in particular special forms of debates.³ In line with Mackenzie's (1989) distinction between reasoning as corresponding to mental processes and reasoning as linguistic, social processes, the main claim of the paper is that the normative remit of logic applies primarily to the second, social meaning of reasoning, as pertaining to dialogical interactions between two or more agents.⁴

I argue for this claim in two ways: by means of an exercise of 'conceptual genealogy' to reveal the historical origins of the (according to the present account, misguided) idea that logic has a normative import specifically for *thought*; and by means of the formulation of an alternative, historically-informed dialogical account of logic. It will become apparent that the conception of logic as tightly connected with *debating* and *disputing* rather than with thinking has been quite influential throughout the history of logic, even though if it is now mostly forgotten.⁵

Naturally, I am not the first to make claims to this effect: the whole research tradition of dialogical logic (Keiff 2009) has been fruitfully exploring the connections between dialogues and logic for decades. I owe much inspiration to some of the authors in this tradition, but I believe that my own dialogical reconceptualization of logic differs from other dialogical proposals in important aspects (in particular with respect to the idea of a built-in Opponent, to be discussed in section 4 below). It is also worth noting that in the context of argumentation theory, the dialogical approaches of pioneers such as Lorenzen & Lorenz and Hamblin remain influential, thus further confirming the fruitfulness of the general approach.

⁴ As also pointed out by Mackenzie (1989, 100), the mental, individual sense of reasoning can then be understood as a special case of the social, multi-agent conception.

I do not need to establish that this conception has been the *predominant* one throughout the history of logic (a claim—which is at any rate almost impossible to prove rigorously, even though it seems in fact very plausible to me). For my purposes, it is sufficient to notice that it has been—quite influential at different times, and in particular in the very early developments in Ancient Greece.

On the basis of this alternative conception of (deductive) logic, the issue can be formulated in different terms, so that new light is shed on some of the difficulties encountered by those who conceptualize the normativity of logic as pertaining to thought alone. In particular, the multi-agent perspective⁶ allows for the formulation of compelling 'bridge principles' between the relation of logical consequence and normative claims, something that is notoriously difficult to achieve in a mono-agent, purely mental setting (MacFarlane 2004; Steinberger forthcoming.). Thus, the historically inspired but essentially *philosophical*⁷ dialogical conceptualization of deductive logic establishes itself as a viable alternative, shedding new light on a number of puzzles and open questions in the philosophy of logic.

The paper proceeds as follows: I start with a summary of recent debates on the issue of the normativity of logic, surveying the positions of authors such as Harman, MacFarlane, and Milne. I then discuss the age-old dialogical conception of logic, and the much more recent association between normativity, logic and thought specifically. Next, I present the details of the dialogical conceptualization of deductive logic that I endorse. Finally, I return to the issue of the normativity of logic, now from a dialogical, multi-agent perspective.

⁶ The 'multi-agent' qualification is required to distinguish the present proposal from 'inner speech' conceptions of reasoning and thinking, which may be conceived of as a dialogue with oneself. In contrast, the present proposal emphasizes the importance of multiple agents and thus the social nature of logic (internalized dialogues with oneself would be the limit cases, not the paradigmatic examples). This being said, the reference to dialogues is not intended to exclude the possibility of polylogues, i.e. linguistic interactions involving more than two participants (Lewinski & Aakhus 2014), but rather to highlight the fact that, on the present account, these interactions take place between two participants, Proponent and Opponent.

⁷ This conceptualization does not depend solely on its historical accuracy to be compelling. It is historically motivated, but remains an independent philosophical thesis.

2. Summary of the debates

The starting point for recent debates on the normativity of logic is the idea that logic has normative import for *thought*, a view often attributed to authors such as Kant and Frege.⁸ For Kant, (general) logic deals with the "absolutely necessary rules of thought without which there can be no employment whatsoever of the understanding." (KrV: A52/B76) For Frege, the laws of logic "are the most general laws, which prescribe universally the way in which one ought to think if one is to think at all." (Frege 1893, 12)

In Change in View, Harman (1986) famously challenged the well-entrenched thesis of the normative import of logic for thought. The main claim is that "logic is neither a normative nor a psychological theory and, although immediate implication and inconsistency may play a role in reasoning, there is nothing special about logic in this connection." (Harman 2009, 333) Logic would be the study of truth-preserving patterns of implication, and by claiming that logic does not have normative import for reasoning, Harman seems to be severing the link between a normative conception of rationality and the concept of necessary truth preservation. In other words, according to Harman, necessary truth preservation would not be an adequate norm for the regulation of one's thoughts and beliefs. He offers a number of arguments to support this claim, such as the principle of 'clutter-avoidance' (one should not clutter one's mind with irrelevant and uninformative consequences of one's beliefs, especially in view of the infinite number of deductive consequences of any sentence), and the observation that, faced with the derivation of a

⁸ But compare (Tolley 2012) on Kant's conception of logic.

⁹ For example, for each sentence A an agent believes, she would have to believe the potentially infinite collection

highly counterintuitive belief P, the best course of action might be to revise the beliefs leading to P as a conclusion rather than simply accepting P.

Although many still seem unwilling to accept Harman's conclusions completely, the consensus seems to be that his observations are not to be taken lightly. A wealth of discussions on the topic thus ensued, including (among others) MacFarlane's (2004) unpublished but widely read and discussed 'In what sense (if any) is logic normative for thought?', Christensen's (2004) *Putting Logic in its Place*, and more recently papers by H. Field and P. Milne in an issue of the *Proceedings of the Aristotelian Society, Supplementary Volume* (2009) (a response to Field by Harman (2009) appeared in a different volume of the *Proceedings*). For the present purposes, it will be useful to focus on MacFarlane and Milne in particular.

Before we proceed, it is worth noting that the use of the term 'logic' in the singular may be misleading, as we now have a plurality of logical systems available: classical, intuitionistic, relevant, defeasible etc. 10 As suggested by Harman's description of logic as the study of truth-preserving implication, what is under debate is above all the property of necessary truth preservation (NTP), which underpins many but not all of these logical systems. If NTP is viewed as a necessary and sufficient condition for the validity of an argument, we obtain classical logic. Intuitionist, relevant and paraconsistent logics, by contrast, have NTP as a necessary but not as a sufficient condition for validity, and this is why

of disjunctive sentences containing A as one of the disjuncts.

¹⁰ Note that what is at stake in the debate between logical monists (Read 2006) and logical pluralists (Beall & Restall 2006) is the idea that there may be more than one *correct* logic, which thus immediately points in the direction of the issue of the normativity of logic(s).

they reject some classically valid truth-preserving principles such as explosion and excluded middle. Non-monotonic, defeasible logics, in turn, deal with arguments that are defeasibly valid, and thus for which NTP is not even a necessary condition for validity. So if what is under discussion here is the NTP property, the whole debate seems to pertain to the logical systems that have NTP at least as a necessary condition, but not to those systems falling outside this range.

With this proviso in place, let us first look at MacFarlane (2004). One of the main contributions of this paper is the introduction of the notion of 'bridge principles' to the debate, i.e. principles that could serve as 'bridges' between two distinct realms such as the realm of logic and the realm of normativity (for thought or otherwise). These are conditionals having a relation of logical consequence between propositions on one side and a statement of normative relations between these propositions on the other side. Bridge principles are useful because they allow for precise formulations of the very idea of logic having normative import for thought, even if the ultimate conclusion to be drawn is that none of them is sufficiently plausible.

MacFarlane takes as his starting point the following schema:

¹¹ And thus, the same considerations apply to what are sometimes described as 'analytic inferences', i.e. those having the property of necessary truth-preservation even if they do not have the property of preservation of validity under substitutivity of non-logical terminology.

The idea of 'bridge principles' generalizes to other cases where an equivalence or analogy is postulated between entities, facts or concepts belonging to different realms. In fact, the term itself seems to have been introduced by Hempel (1966, 72) to refer to principles relating theoretical terms to empirical phenomena, but the general idea is already to be found in e.g. Nagel's *The Structure of Science* (1961).

If A, B => C, then (normative claim about believing A, B, and C)

The instantiations of 'normative claim about believing A, B and C' all take the form of conditionals where the antecedent concerns beliefs about A and B and the consequent concerns beliefs about C. From this general schema MacFarlane obtains 36 different bridge principles, by varying the kind of deontic operation (obligations (denoted by o), permissions (denoted by p), (defeasible) reasons (denoted by r)), polarity of believing (believe (denoted by +), not disbelieve (denoted by -)), and the scope of the deontic operator in the embedded conditional stating the normative claim (only the consequent (denoted by C), the antecedent and the consequent (denoted by B), or the whole conditional (denoted by W)).13

Some of the bridge principles he formulates are: if A, B => C, then...

- (Co-) if you believe A and you believe B, you ought not disbelieve C.
- (Bp+) if you may believe A and believe B, you may believe C.
- (Wr+) you have reason to see to it that if you believe A and you believe B, you believe C.

(Co-) has the 'ought' operator ranging only over the consequent, and has negative polarity ('not disbelieve'). It is more plausible than its positive counterpart, which is exceedingly strong, but it still falls prey to Harman's objection that one should be able to

¹³ "In the naming scheme, the first letter indicates the scope of the deontic operator, the second letter indicates the type of the deontic operator, and the third letter indicates the polarity." (MacFarlane 2004, 8)

revise one's beliefs by retracting one of the premises. (Bp+) has the 'may' operator ranging over both antecedent and consequent, and has positive polarity; it is quite plausible, but it is simply too weak if the goal is to capture the presumed normative grip of logic for thought. (Wr+) has the 'have reasons' operator (which is a defeasible operator) ranging over the whole embedded conditional, with positive polarity.

(Wr+) is deemed very plausible by MacFarlane, but is itself not unproblematic. In particular, given that the deontic operator is a defeasible one, this would imply "learning to live with the idea that logical norms are not, as Broome (2000) puts it, 'strict.'" (MacFarlane 2004, 13) So MacFarlane prefers a combination of (Wr+) with (Wo-):

(Wo-): you ought to see to it that if you believe A and you believe B, you do not disbelieve C.

(Wo-) by itself falls prey to the preface paradox, ¹⁴ whereas (Wr+) by itself does not deliver 'strict' logical norms. How exactly the two principles should interact so as to exert normative force on thought is not spelled out in detail by MacFarlane, which thus means that he does not (and nor does he claim to) offer a definitive answer to the issue of the correct bridge principle(s) relating logical consequence to normativity for thought.

At any rate, MacFarlane is able to narrow down considerably the logical space of plausible bridge principles by showing that most candidates are untenable. Steinberger (forthcoming)

¹⁴ An author rationally believes each of the assertions in her book. But the author also regards herself as fallible, and so rationally believes that the conjunction of all her assertions is false. But if she believes each assertion individually, she should believe the *conjunction* of these assertions.

offers further considerations on the (un)tenability of the different bridge principles, in particular (but not exclusively) with respect to the question of which bridge principle(s) could be used by proponents of paraconsistent logics to argue for their favorite logics on the basis of normative considerations. (Steinberger concludes that no such bridge principle seems to be forthcoming.)

It seems thus that, despite their efforts, these authors see the search for a plausible and sufficiently strong bridge principle connecting the relation of logical consequence to normative claims about believing antecedents and consequents as thus far inconclusive. In contrast, as we will see in section 5, adopting a multi-agent, dialogical perspective will allow for the formulation of very straightforward, eminently plausible bridge principles. The difference will be that the normative claims in question will no longer pertain to thought and belief, but rather to dialogical situations, and moreover that they will involve multiple agents.

In the recent literature on the topic, the only author who does not immediately adopt an individualistic, internalist perspective pertaining to thought seems to be P. Milne:

I'm going to begin with an area in which the normative force of logic is quite clearly discerned, at least by some authorities, and then work back from there to belief. The starting point I have in mind is assertion. (Milne 2009, 269)

By taking assertion as his starting point, Milne places the discussion in the sphere of *speech acts*; although not obviously multi-agent, a focus on assertion should lend itself more readily to the idea of different agents involved in the interaction.¹⁵ His proposed strategy

¹⁵ Indeed, the notion of a speech act is crucial for the present proposal, and as such the proposal has natural ties

is then to make his way back to belief from assertion: "Although there is no quick link between belief and assertion, the dialectical requirements on assertion feed back into normative constraints on those beliefs that constitute one's evidence." (Milne 2009, 269) However, it turns out that the transposition from assertion to beliefs will not be a straightforward matter, in particular in view of the all-or-nothing nature of assertion and the (presumed) probability-like structure of degrees of belief. For our purposes, the exact details of Milne's discussion (in particular the transposition from assertion to belief) will not be directly relevant. Instead, it is one observation he makes at the very beginning of his paper that is particularly significant:

The making of assertions is a rule – or convention-governed practice. Amongst the conventions governing assertion one stands out dramatically for present purposes: that one stands by the logical consequences of what one asserts. (Milne 2009, 269)

In a similar vein, R. Brandom (1994; 2008) maintains that one is *committed* (obligated) to the consequences of one's assertions. Prima facie, this may seem like a plausible (protoversion of a) bridge principle, but it quickly becomes apparent that what 'to stand by' means here requires further clarification. A number of versions of MacFarlane's bridge principles could be formulated for assertion on this basis, all of them with the 'ought' operator:¹⁶ If A, B => C then...

(Co+a) if you assert A and B, then you ought to assert C.

with classical speech act theories such as Austin's and Searle's, among others. What is arguably missing in these classical accounts, however, is the explicit dialogical, multi-agent dimension: a speech act is typically addressed to a putative interlocutor or audience, who will then typically respond with other speech acts, thus giving rise to an interactive situation.

¹⁶ Arguably, only the 'ought' operator can capture the idea of commitment/obligation being transferred over from antecedents to consequents.

- (Co-a) if you assert A and B, then you ought not reject 17 C.
- (Bo+a) if you ought to assert A and B, then you ought to assert C.
- (Bo-a) if you ought to assert A and B, then you ought not reject C.
- (Wo+a) you ought to see to it that, if you assert A and B, then you assert C.
- (Wo-a) you ought to see to it that, if you assert A and B, then you do not reject C.

But surely, it would be ludicrous to demand that one assert every single consequence of what one has already asserted — a linguistic version of Harman's mind-cluttering objection.¹⁸ So it would seem that all positive versions are objectionable on these grounds. As for the negative versions, what exactly does 'not to reject' mean? It cannot mean that one ought explicitly to avow one's non-rejection of C, as this would come dangerously close to asserting C, and thus to the inadequate positive bridge principles. In section 5, I will argue that the best way to make sense of speech-act based discussions of logic, normativity and rationality is to adopt an explicitly multi-agent perspective.¹⁹

'To deny' is also a possibility.

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¹⁷ I use 'to reject' as the counterpart of 'dis-believe', i.e. the act of opposite polarity with respect to assertion.

¹⁸ Which is not to say that either Milne or Brandom would be committed to any of these bridge principles. The point is simply that, when it comes to making the general idea more precise, i.e. in terms of bridge principles, it turns out that things are not as straightforward as they might have seemed from the informal formulation alone. As we will see shortly, it will prove easier to formulate similar principles in terms of proposing and granting rather than in terms of asserting alone.

¹⁹ The dialogical bridge principles to be presented in section 5 are very much in the spirit of what Milne and Brandom propose. The main difference is that I make the multi-agent dimension explicit by formulating dialogical bridge principles involving two agents, Proponent and Opponent.

At any rate, it would seem that, so far, no fully adequate, uncontroversial bridge principle linking the notion of logical consequence, understood as comprising necessary truth preservation, and normative claims about thought and belief has been found. Bridge principles involving the 'may' operator tend to be too weak; those involving the 'ought' operator tend to be too strong; those involving the defeasible 'having reasons' operator do not deliver the kind of 'strictness' that we expect from logical norms (in particular in connection with the property of monotonicity).

Now, given the difficulties encountered within the mono-agent framework, a switch of perspective might provide a novel, potentially fruitful vantage point in this debate. I submit that adopting an explicit dialogical, multi-agent perspective will lead to the formulation of plausible bridge principles involving a necessarily truth-preserving notion of logical consequence. However, the switch to a dialogical perspective would be somewhat arbitrary unless it is independently motivated. Thus, in the next two sections I present a range of independent motivations for the adoption of a dialogical, multi-agent perspective: I argue that logic (as we know it) has an inherently dialogical historical origin (section 3), and that a historically-informed dialogical reconceptualization of logic and deduction makes better sense of a number of otherwise somewhat puzzling characteristics of deductive arguments, and of the notion of necessary truth preservation (section 4).

3. The forgotten dialogical origins of logic²⁰

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²⁰ This section should be viewed as a brief, admittedly superficial survey of the historical developments in question, not as a definitive account; to investigate them in detail would amount to a book-length project. Instead, the role of this section is to motivate historically the shift to the dialogical perspective in sections

The starting point for the reconceptualization of logic and deduction to be defended here is the observation that the conception of logic as pertaining primarily to multi-agent discursive situations of *dialogues and debates*, rather than to mono-agent mental phenomena, has been very influential at different stages in the history of logic. The very birth of logic as we know it in ancient Greece was profoundly marked by the background of dialectical practices (Castelnerac & Marion 2009), and both within philosophy and within mathematics (Netz 1999), the discursive component of *persuasion* was crucial. Netz describes the emergence of Euclid-style mathematics in Ancient Greece in the following terms:

Greek mathematics reflects the importance of persuasion. It reflects the role of orality, in the use of formulae, in the structure of proofs... But this orality is regimented into a written form, where vocabulary is limited, presentations follow a relatively rigid pattern... It is at once oral and written... (Netz 1999, 297/8)

Lloyd (1996) goes a step further and argues that the social, cultural and political context in ancient Greece, and in particular the role of the practice of *debating*, was a necessary (and perhaps even sufficient) condition for the emergence of the deductive method. In effect, the deductive way of arguing seems to have emerged as one approach to argumentation among others, and in fact as a reaction to alternative approaches, e.g. the sophists (Netz 1999, chap. 7).²¹

4 and 5.

²¹ The exact connections between mathematics and logic/philosophy in ancient Greece remain unclear, but there is convincing evidence that some level of communication must have occurred, e.g. recurrent references to mathematics and mathematicians in Plato's dialogues. By and large, mathematicians and philosophers circulated in the same environments (e.g. the early Academy in Athens), and were often the same people. Nothing much for my main claims hinges on whether there was communication between

Interestingly, the transformation from a purely dialectical framework to a framework that is less obviously dialectical, paralleling the transformation from oral to written contexts in mathematics referred to in the passage above, can also be observed in the different logical texts by Aristotle. While his (presumably) older logical texts such as the *Topics* and the *Sophistical Refutations* are clearly about dialogical interactions, in the (presumably) later texts such as the two *Analytics*, the dialectic component is already less pronounced. However, the *Prior Analytics* is full of dialectic vocabulary, and even the definition of a syllogism at the beginning of the text arguably has a dialogical component (as evidenced by the terminology used (Dutilh Novaes ms.)):

A syllogism is an argument in which, certain things being *posited*, something other than what was *laid down* results by necessity because these things are so. (Aristotle, *Prior Analytics* 24b18-20) (emphasis added)

The historical details of the development from pre-Aristotelian dialectic (in particular as documented in Plato's dialogues) to the necessary truth-preservation notion of deduction as presented in the *Prior Analytics* are complex, but it is undisputable that debating practices formed the background for the emergence of syllogistic.²² As for mathematics, the deductive method of proof (which seems to have been developing at roughly the same time (Netz 1999)) received its canonical formulation a few decades later in Euclid's *Elements* (c.

philosophy and mathematics at the time (and the exact directions of influence), but it is a striking historical fact that the deductive method emerged almost simultaneously in mathematics and in philosophy. What matters for our purposes is that in both cases, independently of each other or not, dialectical/dialogical practices provided a crucial background for these developments.

For example, although there are substantial differences between the *Topics* and the *Prior Analytics*, they both offer very similar definitions of a syllogism (*Topics* 100a25-27; *Prior Analytics* 24b18-20). See also (Kapp

1975).

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300 BC), which epitomizes the idea of proving conclusions from a small number of premises and definitions, each inferential step 'resulting by necessity'.

But why the emphasis on 'resulting by necessity'? As an argumentative strategy, a deductive argument is a powerful tool in that it compels the interlocutor to accept the truth of the conclusion if she grants the truth of the premises (that is, given common acceptance of the 'rules of the game'). So it seems plausible that, among different competing approaches to argumentation, persuasion and debate, the deductive approach may have seemed particularly appealing — not only to 'win the debate', but also as inquiry into the truth of the matter under discussion (as noted by Aristotle in *Topics* VIII.5).

It will be argued on conceptual grounds in the next section that a deductively valid argument can be viewed as a *winning strategy* for Proponent: if the conclusion of each inferential step 'follows of necessity', there can be no countermove on the part of Opponent which might block the inference to the conclusion (an idea common to other dialogical conceptions of logic). Whether ancient authors themselves viewed deductive arguments as something along the lines of a 'winning strategy' is a hypothesis requiring further investigation, but the 'resulting by necessity' component must have been quite effective in contexts of debate and persuasion. In other words, necessary truth preservation may have emerged as a *strategic* component in specific adversarial, dialectical games.²³

In the post-Aristotle period, including the Stoics and the tradition of the ancient commentators, logic was predominantly viewed as a tool for argumentation (Bobzien

²³ NTP will then become a *constitutive* principle of the deductive method at a later stage – see section 4.

2006). At later stages, in particular in the Latin medieval tradition, the explicit dialogical component once again took center stage: in this tradition, logic and dialectic (*logica*, *dialectica*) were often treated as synonymous. Generally speaking, oral disputations and debates occupied a central role in Latin medieval intellectual practices (Sweeney 2013; Novikoff 2013), and were extensively discussed in logical texts. For example, one of the main logical topics in this tradition, in particular in the 14th century, was the disputational genre known as *obligations* (Dutilh Novaes 2005). To be sure, not all of Latin medieval logic and semantics was exclusively concerned with disputations and debates, but it is clear that this was a major concern for many of these authors – so much so that the 16th century scholastic author Domingo de Soto still defines dialectic (logic) as "the art or science of disputing" (Ashworth 2011, 129).

Which historical developments led to logic 'losing touch' with its dialogical origins? In the humanist criticism of scholasticism, scholastic logic was one of the main targets, but the humanist authors themselves emphasized the importance of rhetoric (and thus, still focused on public, linguistic events). Instead, I submit that the first significant step in the systematic neglect of logic's dialogical origins corresponds to the early-modern movement towards the inner sphere of thought, as epitomized by Descartes. Commenting on the ideal education for a pupil, he says:

After that, he should study logic. I do not mean the logic of the Schools, for this is strictly speaking nothing but a dialectic which teaches ways of expounding to others what one already knows or even of holding forth without judgment about things one does not know. Such logic corrupts good sense rather than increasing it. I mean instead the kind of logic which teaches us to direct our reason with a view to discovering the truths of which we are ignorant. (Preface to French edition of the Principles of Philosophy, in (Descartes 1988, 186); emphasis added)

Descartes was entirely right in his description of the logic "of the Schools" as a codification of principles for "expounding to others what one already knows". Descartes is after a different logic, a logic of individual discovery: a logic for thinking, not a logic for arguing (an ars inveniendi rather than an ars probandi). But because Descartes makes it clear that he is after a different logic, the dialogical origins of traditional logic are not forgotten or neglected, properly speaking; they are simply rejected. A Cartesian logic of individual discovery was then fully developed in what became known as the Port—Royal logic.

A tendency towards psychologistic, internalist conceptions of logic permeates much of the 17th and 18th centuries (e.g. Locke), and culminates in Kant. Kant provided the most sophisticated version of the (still now) pervasive association between thought, logic and normativity.

As argued by B. Longuenesse (1998), Kant takes as his starting point the question, "what are the a priori conditions for the representations of objects in general?", and appropriates the logic of his time so as to render it useful for his transcendental project. In particular, he turns to the notions of 'judgment', 'form', and 'categories' as found in the logical textbooks of the time, and puts them to use so as to describe the very conditions of possibility for human thinking and perceiving. (To be clear, this is the domain of transcendental logic, which is distinct from his conception of general logic, but transcendental logic presupposes general logic). For Kant, logic does not primarily concern public-sphere

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²⁴ In fact, this holds of most of ancient logic as well, e.g. Aristotle's conception of science in the *Posterior Analytics*.

argumentation; instead, it concerns the inner mental activities of the thinking subject. Moreover, Kant emphasizes the *normative* import of the rules of thought as described by logic. According to him, (general) logic deals with "absolutely necessary rules of thought without which there can be no employment whatsoever of the understanding." (*KrV* A52/B76)

This 'Kantian' conception of logic remained so influential that psychologists were surprised to discover in the 1960s and 1970s that human reasoners do not seem to follow the patterns described by traditional logic in their reasoning (Johnson-Laird 2008). After decades of empirical research on human reasoning, it is now manifest that logic and the canons of deduction do not in any way provide an accurate *descriptive* account of human thinking (Evans 2002; Elqayam & Evans 2011).

Moreover, as argued by Harman, it seems that these canons do not offer an adequate *normative* model for thinking and believing in most real life situations either. Arguably, a normative system which was initially designed to capture phenomena belonging to a certain realm – discursive, dialogical practices of a specific nature, which then gave rise to the venerable deductive method in science and mathematics (e.g. the *Posterior Analytics*) – was at a later stage co-opted and presented as comprising the rules for phenomena belonging to an entirely different realm – the inner, mental realm of thinking.

4. The built-in Opponent conception of logic and deduction

With these historical observations in place, I am now in a better position to present the details of a reconceptualization of the notion of deductive logic that draws inspiration from

its historical dialogical origins. The main claim is that, rather than comprising the canons for correct *thinking*, the traditional principles of deduction reflect rules for engaging in certain kinds of *dialogical practices*.²⁵

Notice that this conceptualization is intended as a *rational reconstruction*, not as a precise historical account of these developments. As such, it is a historically informed but essentially *philosophical* thesis about the nature of (deductive) logic. Deductive arguments would correspond to specific kinds of dialogues, which have both an adversarial and a cooperative component. The adversarial component is what accounts for the property of necessary truth preservation; the cooperative component is what explains the ideal of perspicuity and explanatoriness. Both components will be explained in turn.

The game starts with Opponent granting the starting point, the premises, which are put forward by Proponent: Proponent then puts forward further statements in a step-wise manner, which she claims follow 'by necessity' from what Opponent has already granted. (Additional premises may be conceded along the way as well.) Opponent's role is to verify that this is indeed the case. If it is, he grants the statement just put forward by Proponent; if not, he provides a counterexample, i.e. a situation where the premises are the case but the (preliminary) conclusion is not, thus defeating the statement by Proponent. Another move available to Opponent is the 'why does this follow?' question: if Proponent moves too

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Mercier and Sperber (2011) have recently proposed an *argumentative theory of reasoning*, which bears some resemblance to my proposal. However, they intend their account to apply to all kinds of reasoning, whereas the claim here is that *deductive* reasoning in particular emerged from argumentative practices. There are also important differences between their and my interpretation of the phenomenon known as confirmation bias (Dutilh Novaes 2012a).

quickly from premises to a conclusion, i.e. if she 'skips' intermediate inferential steps,

Opponent may require that the argument be broken down into smaller steps.

At first sight, the participants in these dialogues (Proponent and Opponent) have opposite goals: Proponent seeks to establish the conclusion, while Opponent seeks to block the establishment of the conclusion. It is in this sense that such dialogues are adversarial; Proponent wins if Opponent is forced to grant the conclusion, and Opponent wins if he is able to provide a counterexample to one of the individual inferential steps, thus blocking the establishment of the conclusion. On this level, it is a zero-sum game. However, Proponent's job is not only to 'beat Opponent'; she also seeks to *persuade* Opponent of the truth of the conclusion, if he has granted the truth of the premises. In fact, the goal is not only to show *that* the conclusion follows from the premises, but also *why* it does; this corresponds to the idea that deductive arguments ought to have explanatory value. In this sense, Proponent and Opponent are cooperating in a common inquiry to establish what follows from the premises, and thus to further investigate the topic in question.

This general idea is aptly captured in the following passage by cognitive scientist K. Stenning:

What *follows* in deduction is anything that is true in *all* interpretations of the premises – that is the definition of logically valid inference. Our job, as speaker or hearer of a discourse that purports to be a deduction, is to test the drawing of inferences to destruction, to ensure that what is inferred is true in *all* models of the premises, not just the intended one. It is in this technical sense that logic models adversarial discourse. We may actually be socially co-operating in testing a deduction for validity, and certainly we have to co-operate a great deal to be sure that we are assuming the same things about the range of interpretations which are intended, but there is a core of seeking out all possible assignments of things, not simply finding the intended one. This is perhaps not

accidentally related to the fact that logic arose as a model of legal and political debate. (Stenning 2002, 138)

A crucial aspect of the present proposal is the idea that, in its current incarnation, rather than reflecting actual dialogical practices involving at least two active participants, the deductive method has *internalized* one of the participants – Opponent – in the sense that his role is now built into the framework itself; this is the *built-in Opponent* (BIO) conception of deduction. Every inferential step by Proponent must be immune to counterexamples, so that it is indefeasible even for an ideal Opponent who can contemplate every single situation verifying the premises. BIO differs from earlier dialogical conceptualizations of logic and deduction (e.g. Lorenzen's dialogical logic (Lorenzen & Lorenz 1978), Hintikka's game-theoretic semantics (Hintikka & Sandu 1997)) in that these other frameworks seem to have overlooked the internalized and merely implicit role played by one of the participants, as well as the component of higher-order cooperation (alongside lower-order adversariality).

This insight also finds corroboration in the historical development of the deductive method. As mentioned above, the historical roots of logic and deduction are to be located in the dialectical practices of the early Academy, thus pertaining to actual dialogues involving at least two active participants. But the explicit multi-agent dimension lost its hegemony already in Greek Antiquity, both in logic (see next paragraph) and in mathematics (Netz 1999). Metaphorically, we could say that one of the participants became increasingly silent, until he no longer had an explicit role to play.

Rather than disappearing altogether, Opponent's role of testing the indefeasibility of Proponent's inferential moves and of looking for counterexamples was *absorbed* by the

deductive method itself: deductively valid inferential moves are now only those that guarantee truth preservation. The process of absorption of the role of Opponent by the deductive method is neatly illustrated by the (presumed) historical progression of Aristotle's logical texts. He (presumably) older texts, i.e. the Topics and the Sophistical Refutations, clearly concern actual disputations with flesh-and-blood multiple participants, in the Analytics Opponent has already been by and large internalized by the method. This can be observed in the definition of a syllogism in the Prior Analytics quoted above. A syllogism is still a public phenomenon (the premises are 'laid down'), not a mental process, but the adversarial component and the role of Opponent are now reflected in the property of necessary truth preservation ('results by necessity') which is now an integral part of the method itself: Opponent has been built into the framework.

The claim is thus that the property of monotonicity/necessary truth preservation is a corollary of the adversarial, strategic desideratum of 'beating' the Opponent in a disputation. If there are no counterexamples to each of the steps in an argumentation, then no matter what Opponent comes up with as a counter-move, i.e. no matter what new information he brings in, Proponent's strategy will prevail; it will be a winning strategy for Proponent. The point of a deductively valid argument is precisely that it is valid come what may, and this reflects the idea of an argument being a winning strategy for Proponent, i.e. a strategy which will yield a win no matter what countermoves the other player makes.

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²⁶ It also has antecedents in Plato's notion of an 'inner dialogue', as found in e.g. the *Theaetetus*. Admittedly, a more thorough history of this process of internalization remains to be written, but the progression in Aristotle's logical writings as described here already lends significant support to the thesis, given the centrality of Aristotle for the history of logic. See also (Malink 2015) for further discussion on the differences between the conceptions of deduction in the *Topics* and in the *Prior Analytics*.

Another way of formulating the same general idea is that what starts as a *strategic* but not mandatory component of the dialogical game – putting forward indefeasible arguments – then becomes a *constitutive*, structural element of the deductive method as such: only indefeasible arguments now count as correct moves in a deductive argument.

Moreover, the didactic, cooperative component is now represented by the methodological desideratum that a deductive argument be perspicuous and explanatory, i.e. that each individual step be persuasive and transparent. This is manifest for example in Aristotle's use of syllogistic as a method for scientific justification in the *Posterior Analytics*. Many centuries later, it led to the ideal of proofs whose inferential steps cannot be broken down into even smaller and more compelling inferential steps (as expressed for example by Frege in the preface of the *Begriffsschrift*).

Furthermore, the dialogical setting also provides a rationale for another crucial characteristic of deductive reasoning, namely the fact that only explicitly formulated premises may be used to establish a conclusion. Indeed, Proponent cannot avail herself to external information which has not been explicitly granted by Opponent; it is also the job of Opponent to make sure that Proponent refrains from doing so, thus blocking external assumptions from 'sneaking in' (again, as Frege puts it in the preface of *Begriffsschrift*).

However, it is important to notice that these adversarial dialogues giving rise to the idea of deductively valid arguments are rather contrived forms of dialogical interaction, and essentially restricted to specific circles of specialists. Everyday argumentation does not rely on strict deductive standards; nor does scientific reasoning, at

least not exclusively (it relies equally or perhaps even more on induction and abduction), and perhaps not even argumentation in mathematics (Aberdein 2009). It may well be that, while initially motivated by specific dialogical practices (dialectic), the canons of deductive logic are no longer normative models for any plausible dialogical practices that people currently engage in, outside circles of specialists such as logicians and philosophers.

This might in part explain why untrained participants perform so 'badly' in deductive reasoning tasks, as observed in the psychology of reasoning literature (Evans 2002). It would seem that the ability to argue and reason deductively is not something that arises 'spontaneously' from more mundane argumentative practices. Instead, specific training seems to be required even for a fairly rudimentary level of deductive competence, such as the *modus tollens* rule and *reductio ad absurdum* arguments.

Let me elaborate on this point. First, an important observation is that the majority of dialogical interactions involving humans appear to be essentially cooperative, i.e. the different speakers share common goals, including mutual understanding and possibly a given practical outcome to be achieved.²⁷ True enough, even young children already master the practice of adversarial linguistic interaction (as parents know all too well!); but generally, speakers tend to adopt a cooperative stance when engaging in linguistic interaction.

Moreover, the adversarial dialogues at the origin of the concept of logical necessity are very specific kinds of adversarial dialogues: Proponent proceeds from premises to conclusion in

²⁷ See (Sperber et al. 2010) on the concept of *epistemic vigilance* as ensuring cooperation and truthful communication.

small inferential steps, and each of these steps must be indefeasible in that no countermove by Opponent should be available. A countermove in this case is the presentation of one single situation, no matter how far-fetched it is, where the premises are the case and the conclusion is not – a counterexample. Now, as anyone who has taught introductory logic courses knows, the idea of one single counterexample, no matter how far-fetched, being sufficient to defeat an argument typically strikes untrained students as very odd. This observation is in line with the picture that emerges from empirical results indicating that, in most situations, human reasoning has a strong component of defeasibility (Stenning & van Lambalgen 2008). Human reasoners do not generally operate with arguments and inferences which allow for absolutely no counterexamples (i.e. arguments having the NTP property). Instead, they typically operate with arguments and inferences whose conclusions are true, or at least highly plausible, in cases where the premises are true and nothing abnormal is going on (default reasoning).

The requirement that the conclusion be true in absolutely all situations where the premises are true (including highly unlikely situations) is, for most practical purposes, overkill. Indeed, the emergence of a framework where an argument counts as legitimate only if there is not a single possible counterexample to it, such as the frameworks of classical logic and Euclidstyle mathematical demonstration, creates a niche of specialists, and does not seem to have much bearing on the argumentative and reasoning practices of humans in general (neither descriptively, nor prescriptively, as argued by Harman)²⁸.

²⁸ This is one aspect where Mackenzie (1989) and I part ways. He seems to suggest (much as Brandom (1994)) that the principles of logic permeate 'mundane' dialogical interactions. I tend to believe that the principles of logic instead pertain to rather contrived, somewhat exceptional dialogical interactions, when people systematically challenge each other in their discursive commitments (which, thankfully, is not something we always do when

5. Back to the normativity of logic

At present, virtually no one disputes that classical logic is not a good *descriptive* model of how humans reason (in the internalist, mental sense), but it might still be argued that classical logic (or other logics having NTP as a necessary condition for validity) does provide an accurate *prescriptive* model of how humans ought to think. On this matter, I am convinced that Harman's arguments go a long way towards discrediting the normative claim as well; NTP simply does not seem to be a reasonable norm for thought in most situations, despite initial appearances. One could also add to Harman's original arguments that deductive reasoning is costly and inefficient (a lot of input required for little output); in most cases, the available information underdetermines the deductive conclusion to be drawn in that nothing useful follows 'of necessity' from the available information. The difficulty with finding suitable bridge principles connecting the relation of logical consequence to normative claims about human thought discussed in section 2 is arguably just a symptom of this overall inadequacy.

In sections 3 and 4 I offered an alternative account of what (deductive) logic is normative for, namely specific kinds of dialogical practices, which then gave rise to the deductive method and the concept of deductive proofs in mathematics, logic, and science. I have argued in particular that the property of necessary truth preservation may have emerged from strategic considerations: though initially not mandatory, the formulation of a necessarily truth-preserving argument turned out to be a powerful argumentative tool in

that it constitutes a winning strategy for Proponent. With the absorption of the (ideal) Opponent²⁹ by the deductive method itself, this strategic component then became a *constitutive* feature: now only necessarily truth-preserving inferential moves were permitted (which does not mean that *all* necessarily truth-preserving inferential moves are permitted).³⁰ Hence, a recommendation on how to play the game *well* becomes a condition to play the (new) game *at all*. The origins of the normative import of the property of necessary truth preservation are thus arguably essentially strategic considerations, later turned into constitutive norms.

More important for the present purposes is the fact that the dialogical perspective allows for the formulation of dialogical, multi-agent bridge principles, following MacFarlane's schema ('d' stands for 'dialogical', and the subscripts stand for the agent to whom the normative recommendation applies, i.e. Proponent or Opponent): If A, B => C, then...

- (Wo+do) Opponent ought to see to it that, if he has granted A and B and
 Proponent puts forward C, then he will grant C.
- (Cp+dp) If Opponent has granted A and B, then Proponent may put forward C (and

²⁹ The concept of an *ideal* Opponent is meant to address the worry that validity should not be a matter of an actual Opponent actually happening to stumble upon an appropriate counterexample. The ideal Opponent reflected in the deductive method is the one who will always find a counterexample if there is one available.

³⁰ Indeed, most ancient logicians did not view NTP as a *sufficient* condition for validity: "Ancient logics were all in some sense relevance logics. They insisted that for an argument to be valid, conditions must be met that guaranteed both that it would be impossible for the premises to be true and the conclusion false and that there would be connections of various kinds between the premises and conclusions." (Normore 1993, 448)

require Opponent to grant it).

Why exactly these bridge principles and not others? Regarding the scope of the operator, in (Wo+do) it ranges over the whole conditional in order to allow for the commitment to A and B to be retracted (see below for an equivalent disjunctive formulation which makes this possibility explicit).³¹ In (Cp+dp) it ranges only over the consequent because the permission only affects Proponent.

Why the positive polarity? What about 'not rejecting'? Now, while most frameworks do not draw a distinction between not rejecting and asserting (Restall 2013), there seem to be good reasons to draw such a distinction.³² Not rejecting would also be compatible with suspending judgment about a given C; but given the rules of the dialogical game, what is required from Opponent is that he in fact explicitly grant C if it follows from his previous discursive commitments, and if C is proposed by Proponent.

Why 'ought' as the deontic operator in (Wo+do)? Well, as we have seen with the general description of the dialogical games giving rise to the notion of deductive validity, the point is precisely that Opponent is in charge of verifying whether the statements put forward by Proponent indeed 'follow of necessity' from her previous commitments: if they do, Opponent ought to assent. He is compelled to grant C by the force of necessary truthpreservation. As for (Cp+dp), here again the 'may' modality emerges straightforwardly from the rules of the dialogical game. Given premises A and B, typically there is a multitude

³¹ I owe this point to an anonymous referee.

of different C's that Proponent can put forward; she is not obliged to propose any particular C, but has permission to propose any (or at least most) of them.³³

These bridge principles seem to circumvent the main objections raised against the monoagent bridge principles examined in section 2. In particular, the mind-clutter objection, which was also potentially problematic for the assertion-based bridge principles, does not affect (Wo+do) because Opponent is only required to grant a sentence C if it is explicitly proposed by Proponent. Recall that the mind-clutter objection concerns in particular the exponential explosion of the class of sentences an agent is required to believe (such as, for each sentence A she believes, the potentially infinite collection of disjunctive sentences containing A as one of the disjuncts). In contrast, in a dialogical setting such as the one described here, Opponent must take a stance only with respect to the small, and in any case finite, number of sentences explicitly put forward by Proponent. (Mackenzie (1989, 104) makes a similar point.) One might think that the requirement of logical omniscience remains a problem, but here again the fact that C is put forward by an agent with equally limited cognitive resources alleviates this worry at least to some extent (again, something noted by Mackenzie (1989, 105)). In other words, it is the very concreteness of dialogical situations that make these two objections seem much less compelling.

Notice that (Wo+do) does seem to be contradicted by (a dialogical version of) the preface paradox, just as MacFarlane's (Wo-). But here an interesting dissimilarity between the epistemic context and the discursive context seems to emerge. Recently, a number of authors

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³³ Indeed, considerations of relevance and persuasiveness may also come into play to restrict the range of C's that may be proposed, among those that follow necessarily from A and B.

(e.g. Easwaran & Fitelson 2015) have been arguing that deductive consistency is not a reasonable requirement for ideal epistemic rationality (precisely on the basis of paradoxes of consistency such as the preface paradox).³⁴ In contrast, a stronger case can arguably be made for the plausibility of deductive consistency as a norm within *discursive*, *dialogical contexts*; indeed, it can be argued that consistency (as embodied by the principle of non-contraction) is originally and fundamentally a discursive, dialogical principle rather than an epistemic one (Mackenzie 1989). And if this is right, then a dialogical version of the preface paradox will be much less cogent than the epistemic version. However, for reasons of space this point will remain as a promissory note for future work.

What about the issue of retracting one's original beliefs rather than embracing an implausible conclusion, i.e. the possibility of revision/retraction? To make the possibility of retraction explicit, (Wo+do) can be given a straightforward disjunctive formulation:

• (Wo+dov) Opponent ought to see to it that, if he has granted A and B and Proponent puts forward C, then he will either grant C, or retract his endorsement to A or B.

In practice, this formulation reflects the idea that in such debates, Proponent may intend to show the absurdities that follow from Opponent's commitments in order to force Opponent to revise these commitments. It is precisely the dialogical setting that allows for the introduction of the speech-act of *retraction*, something that a purely logical framework typically cannot easily accommodate.

³⁴ This observation alone would be sufficient to establish the near impossibility of formulating epistemic principles in terms of logical relations between sentences, given the centrality of deductive consistency for most logical frameworks.

It may be thought that these principles provide a disappointingly weak and restricted account of the normativity of logic, one according to which deductive logic has no normative import outside rather specific, highly regimented dialogical situations — and in particular, no normative import for thought in general (as also claimed by Harman). But as Milne, we may now be interested in how (or even whether) these dialogical, multi-agent bridge principles feed back into mono-agent situations of thinking. Now, precisely in virtue of the 'built-in Opponent' component of the present account, it can in fact be generalized to certain mono-agent situations, namely when a given individual is reasoning following the deductive canons by her/himself — say, a mathematician working on a deductive proof. In practice, by following the precepts of the method correctly, the reasoner is playing the roles of both Proponent and Opponent, and this is precisely what makes the deductive method such a powerful reasoning tool in the appropriate contexts — most importantly but not exclusively, contexts of scientific inquiry.

In effect, besides the often cited familiar virtues of deductive reasoning, in particular the high level of certainty it produces (that is, if the premises themselves have a high level of certainty, which is then transferred to the conclusion), I submit that one important but so far underappreciated strength of the deductive method is the possibility of counterbalancing our tendency to hold on to the beliefs we already have. A robust pattern emerging from experimental research on reasoning and cognition is a certain aversion humans seem to have to revising their beliefs, as documented by research on confirmation bias (Nickerson 1998). If well applied, the deductive method can offer a counterbalance to this tendency precisely because a deductively valid piece of reasoning must withstand objections even from an ideal Opponent whose goal is to block the derivation of the conclusion, and who

must become convinced of its truth; it does not allow for 'shortcuts' towards one's own preferred conclusions. Clearly, this is a desirable property for scientific inquiry, as it increases the chances of arriving at non-trivial results.³⁵

6. Conclusions

My starting point for this paper was the still widely held thesis that deductive logic and the property of necessary truth-preservation in particular play a normative role specifically for thought. I surveyed some of the recent literature on this issue, focusing on Harman, MacFarlane, and Milne, and relying on MacFarlane's 'bridge principles'. The conclusion was that there is still no fully satisfying account of the claim that logic has normative import for thought available in the literature, and in fact rather compelling arguments against this claim (Harman 1986). I then offered an alternative account of logic and the canons of deduction, claiming that they have inherently dialogical historical origins. History then provided inspiration for a multi-agent, dialogical reconceptualization of deduction, where a deductive argument would correspond to a specific kind of dialogical interaction between two (fictitious) participants, Proponent and Opponent. The deductive method as such has internalized the role of Opponent, which corresponds to the fact that only necessarily truth-preserving arguments are deductively valid, and that a deductive argument must be perspicuous. This is the gist of the built-in Opponent account of logic and deduction.

I then returned to the issue of the normativity of logic to argue that, rather than thought and

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³⁵ For reasons of space, I refrain from developing this point in more detail here, but the idea is further elaborated in (Dutilh Novaes 2012a). See also (Floridi 2009) on (classical) logical fallacies as 'informational shortcuts'.

belief, logic is in fact a normative codification of specific dialogical practices, i.e. the practices having given rise to the deductive method and traditional logic. This dialogical reconceptualization of the normativity of logic then allowed for the formulation of alternative multi-agent, dialogical bridge principles following MacFarlane's basic schema.

I submit that the present analysis can be seen as a successful application of the built-in Opponent account of logic and deduction to a long-standing issue in the philosophy of logic — the exact nature of the normativity of logic — and thus as an argument in favor of this account. My hypothesis at this point is that a number of other important issues in the philosophy of logic can also be put in a different light once a dialogical perspective is adopted, such as the debate between proof-theoretical and model-theoretical accounts of logical consequence; the nature of the properties of monotonicity, transitivity, reflexivity etc.; the issue of logical pluralism; and many others.

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