Euro Jnl Phil Sci (2015) 5:111–139
DOI 10.1007/s13194-014-0100-y

ORIGINAL PAPER IN PHILOSOPHY OF SCIENCE

Structural realism and the nature of structure

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Received: 28 November 2013 / Accepted: 28 September 2014 / Published online: 9 October 2014

Abstract Ontic Structural Realism is a version of realism about science according to which by positing the existence of structures, understood as basic components of reality, one can resolve central difficulties faced by standard versions of scientific realism. Structures are invoked to respond to two important challenges: one posed by the pessimist meta-induction and the other by the underdetermination of metaphysics by physics, which arises in non-relativistic quantum mechanics. We argue that difficulties in the proper understanding of what a structure is undermines the realist component of the view. Given the difficulties, either realism should be dropped or additional metaphysical components not fully endorsed by science should be incorporated.

Keywords Structural realism · Structure · Underdetermination · Realism

1 Introduction

Ontic Structural Realism (OSR) is one of the most promising ways to develop a form of realism in contemporary philosophy of science. It advances a metaphysical thesis that aims to overcome two of the main difficulties for the realist: the problem of securing reference and approximate truth through *theory change*—the target of the so-called pessimist meta-induction—and the problem of *metaphysical underdetermination*—the fact that the metaphysical nature of the objects posited by certain

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scientific theories is underdetermined by such theories. To solve these difficulties, the ontic structural realist advances a metaphysical thesis to the effect that structures and relations are the fundamental components of the world; objects are secondary—they should either be eliminated or at best re-conceptualized in structural terms (see Ladyman 1998, and French and Ladyman 2003, 2011).

How can the appeal to an ontology of structures save realism given the pessimist meta-induction? Recall that according to the pessimist meta-induction, what in the past were taken to be our best scientific theories are now recognized as being defective; terms that were thought of as having reference in fact do not refer, and theories that were thought of as being true (or approximately true) are now recognized as being false. Similarly, the argument goes, our current best theories will probably have the same fate—sooner or later it is likely that they will also be shown to be false. Thus, it is unclear that one should believe that these theories are true (or approximately true) and that their terms refer. An ontology of structures overcomes this difficulty by allowing for changes in the objects that are referred to in theory change, but insisting that a common structure is preserved through scientific revolutions. That is, in the dynamics of theory change, although the objects referred to by the relevant theories may change, there is *structural continuity* through the coming and going of the theories in question. In the end, we should be realist about structure, not about the posited unobservable objects.

How can the appeal to an ontology of structures save realism given the metaphysical underdetermination? To address this second main motivation for OSR, let us turn briefly to a dispute about the metaphysics of non-relativistic quantum mechanics (see French and Krause 2006, especially Chapter 4). A central issue to be considered is the metaphysical nature of quantum particles. Two options emerge in this context:

- particles as individuals (according to which, roughly, particles have well-defined identity conditions, can be identified and re-identified);
- particles as non-individuals (according to which, roughly, identity is not well defined for quantum particles, there are no identity conditions for them).

These options are, of course, *object-oriented ontologies* (in a broad sense of *object* that does not require well-defined identity conditions for something to be an object).

The main problem for such ontologies in quantum mechanics concerns the fact that the theory, by itself, is unable to determine which option is the right one. So, the argument goes, as far as quantum mechanics is concerned, both ontological options are equally acceptable. According to the proponents of OSR this situation is untenable for a scientific realist: realists should be able to determine the nature of the entities they are realist about (see, for instance, Ladyman 1998, p. 420). Since it is unclear how to do that, given metaphysical underdetermination, one is better off avoiding objects altogether—particularly those whose metaphysical status cannot be determined—keeping commitment only to the structure that is common to both options (French and Ladyman 2003, p. 37). By restricting the commitment only to structure, one can ensure that one's ontology does not overstep what is sanctioned by the sciences.

In both motivations for OSR, the same metaphysical component plays the decisive role: structure is posited as that about which one is realist. In the first case, structure provides the common basis across theory change to anchor one's realism. In the second case, structure allows one to preserve realism in face of metaphysically contentious objects, by providing a common basis among rival metaphysical views regarding the nature of the relevant particles. As a result, one can then resist sliding into anti-realism. For this reason, if there were an adequate account of what a structure is—clearly, a fundamental requisite to make sense of OSR—this kind of realism would be in a privileged situation: it would be able to solve the problems that challenged earlier forms of scientific realism while being clearly and intelligibly formulated.

Before we proceed, we should note that there is a plethora of positions under the heading of *scientific structuralism*, and the same goes for the ontic brand of this family of views. Our focus, in this paper, is on versions of OSR that conceive of objects as either eliminable (a position associated mainly with Steven French) or as ontologically derivative from relations and the structure of which they are a part (a view defended by James Ladyman; see French and Ladyman 2011). Unless otherwise stated, when we write 'structural realism', we mean ontic structural realism of those two specific sorts. This means that versions of OSR allowing for objects as primary entities on an equal footing with relations, such as Moderate Structural Realism (MSR), and other variants that allow for objects as primary entities are not our main target. We aim to examine them explicitly in a future work.[1]

Our aim in this paper is to show that it is unclear that a proper characterization of structure suitable for ontic structural realism can be offered. We argue that there are far too many distinct ways of characterizing structure and relations, and as a result, the combination of realism and a metaphysics of structures becomes, at best, problematic and, at worst, incoherent. We begin, in Section 2, by presenting arguments from a formal point of view. The nature of structures and the representational apparatuses used to characterize them are critically examined. In Section 3, we address the problem of the metaphysical nature of structures and relations. In particular, the ambiguous status of such metaphysical nature is emphasized. We conclude with a discussion of the tenability of combining realism and structuralism. In light of the difficulties of the position, something must go, and the obvious candidate, if we are to keep structures, we argue, seems to be realism.

2 Structure and mathematics

What are the prospects for realism about structures? Within structural realism, we noted, structures play a key role in solving difficulties of traditional realism. Thus, positing such structures may seem warranted. But just what *is* structure? Of course, this question has been raised before. We argue, though, that no matter how it is answered, problems will emerge for the ontic structural realist. In this section, we examine the question in the context of various mathematical representational apparatuses for structures. We divide the section into two parts. In the first, we argue that

[1] A classification of distinct versions of OSR is presented in Ainsworth (2010).

defenders of OSR are ultimately unable to avoid commitment to objects. In the second, we argue that OSR is unable to identify the structure of the world given the diversity of candidates to get the job done.

2.1 Mathematical frameworks and commitment to objects

The adoption of OSR involves two conflicting features, which bring a tension to those who intend to provide a structural realist account of the metaphysics of structures. On the one hand, ontic structural realists argue that theories are better characterized in accordance with the semantic approach, rather than in terms of the syntactic view of theories and related approaches to structure based on Ramsey sentences. In particular, within the semantic tradition, the partial structures approach has been employed to accommodate both 'vertical' relations between scientific theories and data, and 'horizontal' relations among distinct theories (Ladyman 1998; French and Ladyman 2011; da Costa and French 2003).[2] On the other hand, the semantic approach is typically formulated in terms of *set-theoretic* structures.[3] But this commitment to set theory, we argue, introduces objects as key components in the characterization of structures, and is responsible for the tension.

As a framework to define what a structure is, set theory has at least two clear advantages: conceptual clarity and familiarity. It is well known what set-theoretic structures are and how they are constructed: they can be characterized as ordered pairs $E = \langle D, R \rangle$ consisting of a domain of objects and a family of relations among those objects, all of which are found in the set-theoretic hierarchy (see da Costa and Rodrigues 2007 for a general theory of structures). Relations are then defined in terms of the objects that belong to the domain, and not the other way around. Given a structure, the existence of relations, as particular sets, depends on the existence of the elements of the domain: without the objects in D there would be no relations, and, hence, no structure in the set-theoretic sense. This is part and parcel of the iterative conception of set, according to which sets are constructed in stages, and are determined by their elements. Thus, objects are basic in set theory: either sets themselves are objects, such as the empty set in pure set theory and the sets formed from it, or in impure set theory, objects that are not sets—the *Urelemente*—are used to form additional sets, in which case the *Urelemente* are also basic. However, for the reasons discussed above, objects are not allowed as primary entities in ontic structural realism. So, if the structural realist's characterization of structures is implemented in terms of set theory, some maneuver needs to be adopted to defuse the resulting commitment to objects.

To overcome this difficulty one can maintain that structures should be read and understood "from right to left", from the relations to the objects. This would allow for objects to be somehow constituted by, or at least re-conceptualized via, the rela-

[2]For a succinct discussion of partial structures and their application in the philosophy of science, see Bueno and da Costa (2007).

[3]Landry (2007) also highlights the intimate connection between the semantic view and set theory, although her concerns are different from ours.

tions (French and Ladyman 2003, and also French 2010). This strategy is called the "Poincaré manoeuvre" by Steven French (2012, p. 23). According to it, objects are used merely as heuristic devices or stepping stones to obtain the structure. After the structure is characterized, the objects are left behind: either they are taken as metaphysically irrelevant entities or are only conceived as being derived from the relations, depending on the kind of OSR that is assumed. The central point is to ensure that objects are, at best, obtained *after* the relations are given—and obtained from them, not the other way around. Given this maneuver, the need for using objects in set theory to characterize structures poses no threat to a structure-oriented metaphysics. In the end, it is ultimately a matter of knowing how to read the structure, and to realize that any reference to objects to begin with is purely heuristic.

This maneuver, however, faces significant difficulties. First, in set theory, structures are obtained as elements of the set-theoretic hierarchy. As noted, on the set-theoretic account of structure, objects are used to construct relations and structures, not *vice versa* (see, in particular, the theory of structures in da Costa and Rodrigues 2007). The following argument then emerges: (i) Realists about the structure of theories must be realist about the mathematical parts of these theories, since it is not possible to separate their mathematical content from their nominalistic content (see Azzouni 2011). The mathematical content refers to mathematical objects, relations and functions; the nominalistic content does not. Furthermore, (ii) if set theory is used to characterize the mathematical structures in question, sets—as abstract *entities*—will thereby be included among the structural realist's commitments. Thus, a commitment to objects—sets and their members—emerges in the structural realist's metaphysics right from the start. Let's call this argument the "commitment-to-objects argument".[4]

This argument has two important assumptions: (a) It depends on the impossibility of separating the nominalistic and the mathematical content of scientific theories. (b) It also depends on the use of set theory in the characterization of mathematical structures. Let's discuss each of these assumptions in turn.

(a) It is now widely acknowledged that the major attempt at providing a demarcation between the nominalistic content and the mathematical content of a scientific theory—Hartry Field's nominalistic program (see Field 1980)—has not succeeded at establishing the intended result (for a survey and references, see Bueno 2013). And it is unclear which additional resources are available to implement such a demarcation (see Azzouni 2011 for further discussion).

[4]Note that we are not invoking the indispensability argument here, as will become clear below. Our point is that by using set theory, the structural realist is thereby committed to objects—unless a proper nominalization of set theory itself is developed. (But, we will also argue, such a nominalization may conflict with the realist component of structural realism.) Note also that the point goes through independently of how much set theory is ultimately used. So it doesn't matter whether one is dealing with a highly mathematized science or with a less mathematized one. As long as set theory is used by the structural realist (absent a full nominalization of that theory), a commitment to objects emerges.

> Thus, the assumption regarding the impossibility of separating the nominalistic and the mathematical content of scientific theories is one that is reasonable to invoke.

Note, however, that the commitment-to-objects argument is neutral on a stronger claim: the indispensability of mathematics. The claim that scientific theories cannot be formulated without quantification over mathematical objects, relations and functions—which would make these objects, thereby, indispensable to such theories—is not presupposed in the argument. The argument's premises and conclusion are certainly compatible with mathematics being indispensable, but the indispensability is *not required* for the argument to go through. Let's see why this is the case.

As is well known, the indispensability argument aims to establish commitment to objects that are indispensable to our best theories of the world (for discussion and references, see Colyvan 2001). It was originally designed by W. V. Quine (see, e.g., 1960) to force those who are realist about scientific theories to become realist about the mathematics that is indispensably used in such theories. In fact, the argument is supposed to conclude that the grounds that are invoked to establish ontological commitment in science are the same that establish commitment to those mathematical objects and structures that are indispensable to the relevant scientific theories. But the commitment-to-objects argument does not rely on such indispensability. After all, the structural realist's commitment to the mathematical content of scientific theories emerges from the *inseparability* of that content from the nominalistic content of scientific theories, and from the fact that, given realism about the physical world, the structural realist is committed to the nominalistic content—which is, as noted, the content that refers to the non-mathematical features of the world. The commitment to the mathematical content then follows independently of indispensability considerations.

One may argue that the inseparability of the mathematical content and the nominalistic content of a scientific theory just is what the indispensability of mathematics amounts to. But this is not right. We understand the "indispensability thesis" as the claim that scientific theories cannot be formulated without reference to mathematical objects, relations and functions. We understand the "inseparability thesis" as the claim that it is not possible to separate the nominalistic content and the mathematical content of a scientific theory. The indispensability thesis may entail the inseparability thesis, but not the other way around. After all, from the fact that the nominalistic content and the mathematical content of a scientific theory cannot be separated, it does not follow that reference to mathematical objects, relations and functions is indispensable. For a different formulation of the relevant scientific theory can be provided in terms of a different framework in which no reference to such mathematical objects, relations and functions is found. For example, instead of using set theory as the underlying mathematical framework, one can use second-order mereology plus plural quantification (see Lewis 1991, 1993). This framework is committed to mereological atoms (admittedly, a lot of them!), but not to sets. As Lewis shows, as long as there are inaccessibly many mereological atoms, one can mimic the expressive resources of set theory without thereby having the same commitments that set theory has.[5] The important feature is that the commitment-to-objects argument only requires the inseparability thesis, not the indispensability one.

Motivated by these considerations, perhaps the structural realist could try to resist the commitment-to-objects argument by adopting an anti-realist view about mathematics while preserving realism about science. More specifically, maybe the structural realist could adopt a deflationary nominalist view about mathematics (such as the one developed and defended by Azzouni 2004; for some discussion, see Bueno 2013). The deflationary nominalist grants that mathematics is indeed indispensable to science, but resists the conclusion that this provides any reason to be committed to the existence of mathematical objects and structures. This is achieved by distinguishing quantifier commitment (the mere quantification over the objects of a given domain, independently of their existence) and ontological commitment (the quantification that commits one ontologically to the existence of something). If the quantifiers are not interpreted as being ontologically loaded, the fact that one quantifies over certain objects or structures does not entail that such objects or structures exist. It just means that the relevant objects or structures are talked about, that they are objects of thought, as it were. Thus, the structural realist, despite quantifying over set-theoretic structures, need not be committed to their existence, nor to any claim that these structures fully capture the nature of the structures one should be realist about.

The problem with the introduction of ontologically neutral quantifiers in the context of structural realism is that, given these quantifiers, it is unclear how structural realists will manage to specify what their realism amounts to. Unless they provide an independent mechanism of access to, and specification of, the structures they are realist about, the use of ontologically neutral quantifiers will ultimately remove all ontological content from structural realism. It is now left entirely unspecified what, exactly, they are supposed to be realist about. In this way, realism about the physical world seems to have been lost.

Perhaps structural realists could insist that the structures they are realist about are those that were obtained via inference to the best explanation as part of the success of science. Mathematical structures only *represent* the nominalistic (physical) content, which is the content structural realists are ultimately committed to; they need not be committed to the mathematical content. In other words, the set theory that structural realists invoke only play a representational role; it does not provide any guide to the commitments structural realists have.

However, with this response, the initial problem simply returns: How can the nominalistic content be specified without a proper nominalization of mathematics in the first place? If quantifiers are *not* ontologically neutral, given the use of set theory structural realists are committed to objects (namely, sets), which is incompatible with

[5] One may worry about the full success of Lewis' construction. Since the notion of inaccessibility is fundamentally set-theoretic in nature, aren't sets still presupposed (Bueno 2010)? Even if the proposed reconstruction is expressively equivalent to set theory, is it in fact as effective for the formulation of empirical theories as set theory is? These are fair concerns, but they are also beside the point in this context. The purpose of the Lewis example is just to make a conceptual point, namely, that the inseparability and the indispensability theses are not the same. We need not argue that the indispensability thesis is in fact false; only that it *can* be.

their insistence that structures, rather than objects, are fundamental. Alternatively, if quantifiers *are* ontologically neutral, it is unclear how structural realists can specify what they are realist about, since such quantifiers will remove all ontological commitment from what is quantified over—even if one quantifies over what was obtained, by means of inference to the best explanation, on the basis of the success of science.

Perhaps the structural realist could maintain that true existential statements that follow from our best theories indicate such ontological commitment. But with ontologically neutral quantifiers in place, this suggestion would not be enough to express ontological commitment, since these quantifiers only indicate that some part of the domain is being considered, not that what is being quantified over exists. An existence predicate needs to be introduced for that. But what should the content of this predicate be?

One possibility is to propose that the existence predicate expresses ontological independence: those things that are ontologically independent from our linguistic practices and psychological processes exist (Azzouni 2004). There is, however, significant disagreement in discussions of realism in science about what kinds of things are (or are not) ontologically independent from us. Standard scientific realists who are committed to the existence of quantum particles insist that these particles are ontologically independent from us. Ontic structural realists resist that commitment, insisting that ontological commitment to things of such dubious metaphysical status should be avoided. If these realists about science are also platonist about mathematics—in particular, about mathematics used in science—they will insist that mathematical structures exist, given that these structures are ontologically independent from us. In contrast, if these realists are nominalist about mathematics, they will point out that, since mathematical structures are not ontologically independent from us—we made them up, after all—these structures do not exist. It is, thus, unclear that ontological independence is of much use in such ontological debates.

But perhaps the structural realist may respond by noting that the appropriate existence predicate should identify a suitable mechanism of detection of the relevant structures. After all, it is only with such a detection mechanism that the relevant mathematical structures (suitably interpreted) can have any empirical significance. If, however, there is such a detection mechanism, the burden is now on structural realists to describe it, show how it functions, and specify precisely how such mechanism yields a stable account of the nature of the structures they should be realist about. It is only after this is done that their view would secure the relevant realist content. But the difficulty is to ensure that the usual mechanisms of detection (such as various scientific instruments used in scientific practice) detect structures rather than particular objects. Consider the micrograph from an electron microscope. It may be argued that on the surface of that image we find the representation of particular objects: whatever objects that were present in the sample when the micrograph was generated. Rather than a commitment to structures, on this view, micrographs provide information about the relevant objects. The worry is that structural realists may end up presupposing objects as part of the specification of whatever detection mechanisms they invoke.

In response, structural realists could argue that micrographs do exhibit structural features: the various relations among the objects that are represented in the image.

Moreover, they continue, those structural features correspond to structural components of the world. But it is unclear that this response is really open to structural realists. Micrographs can certainly display structural traits, but how can structural realists make sense of these traits if they are formulated in terms of relations among objects in the sample? As an illustration, consider a micrograph produced by a transmission electron microscope, which represents ribosomes in a cell. The micrograph represents the ribosomes as located in a particular region of the cell, say, near the membrane. It also represents them as bearing some spatial relations to other ribosomes and other cellular components. We can grant that these features are structural: they display relations among objects, after all. However, in order for the features to be structural, ribosomes need to be taken as objects rather than structures: a structural understanding of ribosomes is obtained via the relations they bear to other cellular components. But this means that ribosomes, as the terms in the various relations, are ultimately understood as objects. As a result, objects are ultimately presupposed, and we end up with an approach that ontic structural realists are unable to embrace.

The advocate of ontic structural realism may respond by arguing that, for conceptual considerations, researchers may need to introduce objects, which bear a variety of relations, at certain stages of their inquiry in a particular field. The ribosome case is not different. However, once ribosomes are properly considered, they are best understood as involving a plurality of relations that hold between items provisionally postulated as objects, that is, as relation-bearing items.

However, this means that ribosomes are ultimately conceptualized as objects, so that they can be relation-bearing items. It doesn't matter whether the reasons for this are conceptual, empirical, or something else entirely. Postulating objects is not an option for those structural realists who insist on the elimination of entities.

But perhaps structural realists could insist that the usual mechanisms of detection ultimately allow us to detect properties and relations (presumably of the relevant objects). Access to detection properties (see Chakravartty 2007) can be forged by scientific instruments. And by combining access to such properties and the relevant relations, access to a particular structure emerges. In this way, it is specified what the structural realist is committed to. It is unclear, however, that this move will help structural realists, since the proper characterization of detection properties also ultimately presupposes objects—the objects that have the relevant properties. As a result, structural realists would simply be back to where they started.[6]

(b) The commitment-to-objects argument also relies on the (widespread) use of set theory to characterize mathematical structures. Perhaps this argument—as well as the Poincaré maneuver—could be resisted by simply rejecting such use of set theory. We argue, below, that problems will emerge even if set-theoretic structures are not invoked. For the moment, note that the rejection of set theory comes with a significant cost for the structural realist. To begin with, recall that

[6]More generally, one of the crucial features of Anjan Chakravartty's semirealism (Chakravartty 2007) is to argue that realists need the commitment to both objects and some properties and relations—and, thus, some structures—in order to get off the ground. Clearly, given the commitment to objects, this is not a move open to ontic structural realists.

> an alleged virtue of the semantic approach is that it does not take one's theorizing about the sciences too far from actual scientific practice (as the syntactic approach arguably does; for an overview, see Suppe 2000). So, to avoid contradicting scientific practice and its widespread use of set theory, the structural realist who also adopts the semantic approach had better preserve the usual way set-theoretic structures are formulated and introduced in actual scientific practice. It would be disingenuous to dismiss the use of set-theoretic structures as irrelevant at this point. The way mathematicians and physicists introduce and formulate structures should be taken seriously in this context too. The result, however, is a commitment to objects as part of the resulting metaphysics.

The structural realist may insist that set-theoretic structures only provide representational devices regarding the structures in question. One should not read off anything about the fundamental nature of the structures one should be committed to from the mere fact that they can be represented set-theoretically. If set-theoretic structures presuppose objects, so be it. This simply shows that these are not the structures the ontic structural realist is ultimately realist about.[7] A similar view is advanced by Brading and Landry in a series of papers (see Brading 2006, 2011 and Landry 2007). According to them, set theory plays no special role in characterizing structure and, in particular, in articulating the notion of *shared structure*, a central notion for any version of structuralism. Their suggestion is that this notion can be left unspecified (that is, it should not be assumed that it is a set-theoretic notion to begin with), and its nature should be decided on a case-by-case basis. All that matters is that we have a notion of shared structure.

These responses, however, have a cost. Without the specification of the nature of the structures that the ontic structural realist is realist about, the very content of OSR is left unspecified. It then becomes unclear about what, exactly, the structural realist is realist. Without a clear characterization of the structures in question, the view ultimately lacks content. Thus, in order for OSR to get off the ground, a proper specification of structure is required. Furthermore, to advance, as Landry (2007) does, that the context determines the kind of characterization of structure required in each case falls prey to two difficulties. First, if the available options involve objects (as Landry seems to allow), then those who don't want to be committed to objects in the first place are not better off. Second, if the notion of structure is left unspecified, then one is left in the dark as to what one's realism is about. None of the options seem palatable to the OSR-theorist.

But perhaps the structural realist could suggest that the specification of the relevant structures is done via ostension. Maybe there is no way of determining the scope of one's structural realism but by pointing to particular instances of the relevant structures about which one is a realist. The problem with ostension is that, for familiar Quinean reasons, it is radically indeterminate. One may point to an inscription on a piece of paper that represents, say, a set-theoretic structure, and state "I'm realist about that". But what does 'that' refer to? The piece of paper? The inscription on the

[7] This line of response has been suggested by Steven French and James Ladyman in conversation.

paper? The representation that is conveyed by the inscription? The object that is represented? The content of the representation? The physical interpretation associated with that content, and if so, which among the various such interpretations does one pick out? And how, exactly, can any such interpretation be picked out by ostension? Clearly ostension is entirely inadequate for the task at hand.

One could try to avoid the commitment to objects by shifting from classical set theories to a non-classical set theory, such as quasi-set theory (for an exposition, see French and Krause 2006, Chapter 7). As is well known, quasi-set theory allows for collections of things that lack identity conditions, the non-individuals. It is, thus, crucial for quasi-set theory that the extensionality axiom of classical set theories does not hold in general. After all, this axiom specifies identity conditions for every set, thus ruling out, by fiat, things that lack identity conditions: sets x and y are *the same* just in case they have *the same* members. The main motivation for introducing things that lack identity conditions is to model the behavior of non-individuals in quantum mechanics, according to the interpretation of the theory that admits of such things. Moreover, it is possible to define structure in quasi-set theory too, so that the elements of the domain could now be taken as being non-individuals.

Given the restriction on the scope of the extensionality axiom, it may be thought that quasi-set theory could avoid the commitment to objects. Does that alleviate the burden on OSR? Not really. Even though some philosophers have advanced the idea that quantum mechanics with non-individuals is a version of OSR (in particular, see Votsis 2011), that is still an object-oriented ontology. Non-individuals, as understood in quasi-set theory, are objects: one quantifies over them; they have certain properties (and lack others), and they bear relations to other things. As French (2010, p. 94) makes clear, OSR does not get rid of the *individuality* of particular objects, it gets rid of *objects* altogether, whether they are individuals or not. This is important, since metaphysical underdetermination between the metaphysical packages of individuals and non-individuals is one of the main motivations for OSR. So, to adopt an alternative metaphysical package by allowing a set theory with non-individuals should not be seen as softening the burden for OSR. Non-individuals are objects too—to take this path is ultimately to accept commitment to objects.

2.2 A plurality of structures

Another significant difficulty for OSR, and for the Poincaré maneuver in particular, is that even if the latter managed to avoid commitment to objects in the characterization of set-theoretic structures, it is open to an important kind of underdetermination: it involves distinct but elementarily equivalent structures that are models of the same theory (Bueno 2011).[8] Due to the upward Löwenheim-Skolem theorem, first-order theories with models with infinite domains have elementarily equivalent but non-isomorphic models for every cardinality. The models are importantly different (since

[8]Building from an argument advanced by Bueno 2011, this section examines additional considerations regarding the philosophical significance of elementarily equivalent but non-isomorphic models to the OSR debate.

they are non-isomorphic), but exactly the same first-order sentences are true in them (since they are elementarily equivalent). Which of those many models represents the structure of the world? That is, which of this huge number of structures is the structural realist realist about? An account of how one can choose among such structures and determine the right one needs to be offered. But it is unclear how this could be done. On what epistemic grounds can a structure be preferred over another that is elementarily equivalent to it? It seems that there is no simple, epistemic way to determine which particular structure is that of the world.

Perhaps the choice among the various structures can be made based on pragmatic considerations, that is, considerations related to the users of the theory rather than based on epistemic, evidential grounds (see van Fraassen 1980). Pragmatic considerations include simplicity, familiarity, fecundity, and expressive power (the usual theoretical virtues). They provide reasons to prefer certain structures over others. It is undoubtedly easier to work with simpler, familiar structures, which are also fecund and have rich expressive power. However, this is a reason to *accept* the structures in question rather than *believe* that they properly describe the world (see van Fraassen 1980). After all, absent some metaphysical principle according to which the world itself is simple (in some sense), or that structures that are familiar, fecund, and rich in expressive power are more likely to describe reality than unfamiliar, barren, and inexpressive ones, pragmatic reasons alone are not sufficient to support the conclusion that the chosen structure is correct. Thus, a choice on purely pragmatic grounds is unable to support the realist component of the view. For if we were to choose pragmatically what the structure of the world is, we would not thereby have grounds to believe that such a structure is right. As a result, with multiple non-equivalent structures available, and no epistemic reason to choose between them, a case of underdetermination arises for the metaphysics of structures underlying OSR. In the end, it is unclear that the structural realist has the resources to specify the particular structure one should be realist about.

But perhaps there is a way out here; one that is usually invoked in the defense of the superiority of the semantic approach over the syntactic view. Only the *intended* models of the theory in question are picked out. The fact that the semantic approach can accommodate this move is an important benefit of the view and a significant reason to prefer it over the syntactic approach (see Suppe 2000). However, this way out is not open to the structural realist. How is the choice of the *intended* model supposed to be made? Once again, to invoke pragmatic considerations as the basis to determine the nature of reality is not an available route. What is required is a structural, epistemic constraint on the choice of the structure of the world. But which structural, epistemic constraint could be invoked in the choice of the intended model? One would need to have independent reasons to believe that the fact that the intended model is *intended* somehow makes it more likely to be the right one—the one that corresponds to the structure of the world. But no reason has been provided as to why this is the case. And it is unclear that there is such a reason available to the structural realist. It simply begs the question to assert that the intended model is natural, in the sense that a natural model provides the correct description of the structure of the world. Moreover, if by 'natural' it is meant that the relevant models capture natural kinds, it is not obvious that such a move would be open to the structural realist either. For the postulation of natural kinds introduces an ontology of objects—those that have the relevant kinds—and that is precisely what the ontic structural realist is trying to avoid. Alternatively, if kinds are identified extensionally, in terms of the sets of objects of the relevant kinds, the concerns raised earlier about the ontological commitment to sets—which are ultimately objects, after all—arise again.

A further problem prompted by the existence of elementarily equivalent non-isomorphic models concerns the very idea of re-conceptualization of objects. Recall that for the kind of OSR we are considering here, objects are derived from structures, they are either contextually individuated or merely the nodes in a web of relations. But even supposing that we could somehow fix a common underlying structure among those non-isomorphic models, there would be trouble with the number of objects that such a structure gives rise to. If we are going to take seriously the claim that objects are nodes in the web of relations or that they are individuated contextually by the relations of the structure, the cardinality of objects obtained in this way should be fixed. That is, one would expect that the structure of the world should give rise to one world, which has a well-determined number of objects (exactly the number of objects in reality), even if objects are to have only a secondary metaphysical status. However, due to the argument above, the same theory may give rise to structures with distinct domains, of distinct cardinalities. Using the vocabulary introduced above, reading a structure $\langle D, R\rangle$ 'from right to left' may be performed in several distinct ways, each of them giving rise to a set D of distinct cardinality, and each of these sets could be the domain of a model of the theory and, thus, each could claim rights to be the one that properly represents reality.

The structural realist may complain that to assume that there is a well-determined number of objects in the world is too stringent a requirement. It is not possible to determine that number without providing individuation conditions for objects. And due to vagueness, indeterminacy, or intractability, it may not be possible to determine what that number is. Let us grant this point. Despite that, presumably the structural realist who is willing to allow for a reconceptualization of objects in terms of structures also allows for there being some number of objects in the world. The determination of that number need not be made sharply. Perhaps the structural realist only indicates that the relevant number is within a certain range. However this determination is implemented, the problem just raised will arise again. For sets of distinct cardinality would emerge from reading the relevant structures 'from right to left', and each of these sets could be used as the domain of a model of the theory that represents the world—as long as the cardinality of the domains is within the specified range. Alternatively, if no range at all is specified, then it becomes unclear why the structural realist intends to re-conceptualize objects in terms of structures. If there is no number of objects in the world, if not even a range for that number can be provided, the structural realist seems to lack a reasonable motivation to introduce such objects in the first place.

Before we proceed, we should make it clear that the previous arguments are not a restatement of the well-known *Newman objection* presented to epistemic versions of structural realism. According to the Newman objection, attempts to articulate the theoretical content of a scientific theory (such as through its Ramsey sentence) fail to specify the precise extension of the theoretical relations. In fact, given any set with the *same cardinality* as the intended model, we may convert that set into a model of the theory (see Ladyman 2013 for general discussion).[9] Our point, in contrast, focuses on the difficulties that non-isomorphic, but elementarily equivalent models—which, thus, have distinct cardinalities—raise to OSR; it goes in the opposite direction than Newman's. While Newman's objection moves from collections of objects with the same cardinality to relations, we go from relations to collections of objects with distinct cardinalities. Since the relationship between objects and relations in OSR is supposed to be such that the former are 'derived' from the latter, our argument shows that such an operation, however implemented, can be executed in a plurality of ways. No structural constraint determines a particular domain as the correct one. As a result, this is not a version of Newman's objection. In Section 3, when we examine metaphysical characterizations of the relationship between structures and objects, we argue that additional difficulties emerge as well. But, once again, the argument proceeds from relations to objects, not from objects to relations.

Perhaps that problem of the existence of multiple structures can be overcome if we use a higher-order logic.[10] With second-order logic we obtain categoricity for important mathematical theories, so that non-standard models are avoided in those cases. However, there is a price to be paid, and it is unclear that the desired result can be reached. First, as is well known, categoricity for higher-order theories only obtains when what is called *standard semantics* is taken into account, that is, a semantics in which the higher-order variables for properties and relations run through the whole plethora of properties and relations available. However, when *Henkin semantics* is employed, that is, the one in which variables run through some (but not necessarily all) subsets of the whole domain of relations and properties, non-standard models appear again, and a version of the Löwenheim-Skolem theorem holds. Even if we could reasonably choose only standard models (that is, models invoked in standard semantics) in a way acceptable to the structural realist, there would still be difficulties: (a) It is not clear that our best empirical theories are categorical, so the problem of determining what the right structure is would not be avoided. (b) Higher-order logics using standard models are incomplete. And it is unclear how structural realists can accommodate such incompleteness. Which status should they assign to statements that are true but not derivable from the relevant principles? (c) Objects are an integral part of the formalism of second-order logic, in the sense that any interpretation of such formalism—whether in set theory or in some other formal framework—presupposes objects. So, in the end, the OSR-theorist doesn't solve the problem by shifting to higher-order logics.

A different proposal concerning the relation between objects and structures recommends the use of category theory instead of set theory (see Landry 2007 and Bain 2013). It is argued that category theory is better equipped to deal with the elimination of objects because categories are not defined in terms of objects, but rather in terms of morphisms (or arrows). There is no need to appeal to any kind of *maneuver*

[9] Demopoulos (2003) also discusses this worry, and he links it to Putnam's model-theoretic argument and to the semantic view of theories, but it is independent from the concerns we raise here.

[10] For an excellent discussion of second-order logic, see Shapiro (1991).

here: objects are already given a secondary place. So it seems that category theory deals more adequately with the elimination of objects required by OSR and provides a better representational system for the view.

One worry with this proposal is that the choice between set theory and category theory is being made on pragmatic grounds, given the expressive resources of category theory and those of set theory. But it is unclear why having certain expressive resources, such as being able to formulate structures without presupposing objects, is sufficient to ensure a realist reading of the categorial framework—as the one that provides the proper characterization of the structure of the world. One would need to offer reasons as to why such a pragmatic choice will deliver structures that properly describe the world—something that is needed given the intended realism about structure. However, in light of the considerations made above, it is not clear that pragmatic reasons, such as the expressive resources of the categorial approach, are good epistemic guides: they may provide reasons to accept the category-theoretic framework, but these need not be reasons to believe that the framework is true, or likely to be so (see van Fraassen 1980).

The category theorist may respond by noting that the adoption of category theory is not done on pragmatic grounds: set theory is just inadequate to represent objectless structures, and so it fails to express properly what needs to be expressed. Category theory, in turn, is adequate to the task at hand. Thus, its adoption is not made on the basis of pragmatic considerations, but emerges from the adequacy of the expressive resources of the theory itself. However, is category theory really adequate in the relevant respect? We don't think it is. After all, the definition of a category *presupposes objects*. A category is defined in terms of objects and arrows (see Awodey 2010, pp. 4-5):

- For each arrow, there are objects, the domain and the codomain of the arrow.
- For each object there is an arrow (the identity arrow of that object).
- Given two arrows such that the codomain of one is identical to the domain of the other, there is an arrow which is their composite.
- The composition of arrows is required to be associative (that is, the composite of the composite of arrows f and g and the arrow h is identical to the composite of the arrow f with the composite of the arrows g and h—as long as f's codomain is identical to g's domain, and g's codomain is identical to h's domain, so that the relevant compositions are defined).
- All arrows are required to have a unit (that is, for all arrows f, the composite of the identity arrow of f's domain and f is identical to the composite of f and the identity arrow of f's codomain, and both such compositions are identical to f).

Clearly, identity is presupposed throughout this definition: in particular, in the characterization of the composite arrow (which presupposes the identity of the domain of an arrow and the codomain of another), as well as in the formulation of associativity and unit (both of which presuppose the identity of the relevant arrows). Thus, genuine objects are presupposed: one quantifies over them, they have certain properties (e.g., each object has an identity arrow) and lack others (e.g., an object can be distinguished from an arrow), and they bear relations to other objects and arrows (some objects are domains of an arrow and codomains of another arrow, others are not).

Thus, given this definition and the crucial role played by objects in it, in the absence of objects, a category cannot even be formulated. As a result, category theory does not seem to provide a better alternative than set theory does *vis-à-vis* characterizing structures without objects.

Another concern regarding the adoption of category theory, raised by Steven French (2012, p. 24), is that category theory is just too abstract to provide the proper replacement for the traditional set-theoretic tools that are needed for the semantic approach. For example, set-theoretic resources are readily available to characterize relations between theories—thus expressing structural continuity in scientific change—while category theory seems better equipped to deal with types of structures. French suggests that one could perhaps use the resources of category theory and set theory interchangeably, according to one's needs: when dealing with types of structures, appeal to category theory is required, while when it is relations between theories that are being dealt with, then set theory should be used. However, once again, the trade-off between the two frameworks is performed at the pragmatic level, and it is unclear whether this satisfies the structural realist's needs.

Finally, as noted, given that categories are defined in terms of arrows (morphisms) *and objects*, category theory is not a framework that an OSR-theorist can adopt to answer the question regarding the nature of structures. Similarly to set theory, it is ultimately an object-oriented view.

Those arguments can also be directed against Bain's claim that since set theory introduces *surplus elements*—the objects in the domain of the structures—category theory should be preferred because it eliminates such surplus components (see Bain 2013). However, the idea that surplus elements should always be eliminated goes against OSR, since such elements, in the form of surplus structure, are explored as heuristic devices in scientific discovery (see da Costa and French 2003). Moreover, if the surplus elements are restricted to objects, Bain's proposal begs the question against object-oriented realism. The claim that we should choose the formal framework that removes objects (for it helps us to get closer to the truth) is acceptable only if we are already converted to the credo that objects are secondary or eliminable.

Ontic structural realists may respond by insisting that this criticism is raised at the wrong level: surplus structural features, if explored as heuristic devices in scientific inquiry, are invoked at the level of the representation of epistemic resources rather than at the level of the structural features of scientific theories, which is the relevant one as far as ontological commitments are concerned. In response, we certainly grant the distinction between the representation of the epistemic status of certain theories within scientific practice (which typically involves some philosophical reflection about the practice) and the theoretical resources invoked by scientists to solve problems (which is the proper scientific domain, in which ontological commitments are articulated). However, by invoking the role of surplus structure in scientific reasoning, ontic structural realists are focusing on how such surplus is used as heuristic devices in scientific discovery, and thus such surplus structure is at the level in which ontological commitments are expected to be found.

With regard to the charge that Bain's proposal begs the question against object-oriented realism, structural realists will note that they provide independent reasons to avoid commitment to objects (Ladyman 1998; French and Ladyman 2003, 2011 and Ladyman and Ross 2007). Thus, they conclude, preference for an objectless framework does not beg the question. It is just an expression of the appropriate framework in which to formulate and develop structural realism. The problem with this response, in the context of Bain's defense of category theory (as opposed to set theory) as the proper framework for structural realism, is that the elimination of objects is ultimately incompatible with category theory: as argued above, the formulation of a category presupposes objects and cannot be implemented without them. Thus, despite the reasons structural realists provide to avoid commitment to objects and Bain's categorial proposal, category theory does not yield the appropriate framework in which to articulate an objectless structural ontology.

Furthermore, if one accepts that to get rid of surplus structure is part of the business of getting closer to the truth, then the metaphysical underdetermination—one of the main reasons to adopt OSR—does not emerge. Indeed, as Redhead and Teller (1991) have argued, Hilbert space structures employed in quantum mechanics introduce surplus structures (vectors without the adequate symmetrization) that allow for the entities in the theory to be interpreted as individuals. Their advice is to eliminate such surplus structure shifting to a Fock space formalism. That move would allow us to keep non-individuals (with no metaphysical underdetermination holding anymore). As a result, OSR would lose one of its main motivations. Thus, a category-theoretic approach is either not required or has to dispute priority with non-individuals.

Finally, one may wonder about the metaphysical status of category-theoretic objects: are they individuals or non-individuals? However this question is answered, the resulting framework will make an assumption about the nature of objects that conflicts with the metaphysical underdetermination that is so crucial for the OSR-theorist.

To overcome these difficulties, one could adopt a pluralist approach: to accommodate relations between theories one could employ a set-theoretic framework, but to explore the consequences of modern physics to the concept of objects we shift to category theory (French 2012, p. 24). On this view, the best of each framework would be used in accordance with the needs. However, how does this pluralist and pragmatic stance fit with realism? If one cannot discern precisely the boundaries between the mathematics and the physics—on the structural realist picture, they are often intertwined in the descriptions of what goes on at the fundamental level, their boundaries blurred—and given the commitment to realism, then some form of realism about the mathematics will have to be adopted. But this pluralist approach seems to be in tension with realism, since it fails to deliver a clear ontology. On this approach, ontological commitments shift between sets and categories, but these ontologies are fundamentally different: one gives priority to objects (set theory) the other to arrows/morphisms (category theory). Furthermore, the fact that both are candidates to represent the structure of reality yields another form of underdetermination for the realist: one cannot decide which of them (if any) properly represents the nature of the world just by looking at our best scientific theories. However, OSR was designed precisely to avoid this kind of underdetermination, keeping the commitment to whatever structure was common among the conflicting theories in science. Unfortunately, no such structural communality is available here, given the differences between sets and categories.

It may be objected that the ontic structural realist need not be a mathematical platonist, and that nothing in OSR requires the ontological commitment to mathematical structures. Furthermore, the argument goes, the blurred boundaries between the mathematics and the physics in certain contexts of modern physics—such as when symmetry reasoning is involved—is a fact of the science, not a feature of a structuralist view. In the end, it is far from obvious that realism about the structures employed in modern physics entails realism about the structures of any particular branch of mathematics.

In response to the point that OSR does not entail mathematical platonism, the situation is more complex than it may initially appear. On the surface, it may seem that the two views are independent from one another. After all, OSR is a form of realism about the (fundamental) structure of reality. As such, it seems to make no claim about the existence of mathematical structures—which is the scope of a structuralist version of platonism (that is, a form of realism about mathematical structures). But, in fact, if the mathematical content of a theory cannot be separated from its physical (nominalistic) content (Azzouni 2011), it is unclear how the structural realist can restrict ontological commitment only to the physical content without having first already nominalized mathematics. And as we argued above, by nominalizing the mathematical content via ontologically neutral quantifiers, the physical content will end up being nominalized as well—unless some independently motivated detection mechanism is provided. But none has been by the ontic structural realist.

With regard to the point that it is a fact of the science that the boundaries between the mathematics and the physics are blurred (rather than a feature of the structural realist interpretation of it), it should be noted that, whatever the source of that fact ultimately is, structural realists explore and emphasize it, insisting that standard forms of scientific realism are unable to properly accommodate it. If in the end structural realists are similarly unable to make sense of this fact properly, a significant challenge for their view results.

Even if one could reasonably overcome these difficulties, there would still be a related problem to be solved: distinct formal apparatuses may be employed for the same purpose in non-equivalent ways. As Bain (2013) notes, this is an instance of what is now called 'Jones Underdetermination': the same theory has distinct formulations encompassing distinct ontologies. However, there is only one structure of the world (according to the realist component of OSR), and it is this structure structural realists are realist about. How can the underdetermination among the various mathematical frameworks be overcome? To avoid the above pluralism, Bain (2013) recommends assuming naturalism and semantic realism: we accept physics at face value, and agree that it speaks about objectless structures. However, it is not clear that semantic realism and naturalism entail OSR: on a different view, they would motivate an ontology of non-individuals, since non-individuals are also posited in significant interpretations of non-relativistic quantum mechanics. Indeed, it is hard to understand how semantic realism and naturalism can solve the problem of determining uniquely the relevant ontology. More should be said about how to extract from physics such a commitment for OSR if we are not to end up with just another option for underdetermination.

Suppose that OSR is reformulated so that the main goal of the view is to provide a coherent ontology for fundamental physics, thus going beyond the concern with the individuality of quantum particles. On this formulation of OSR, the world is fundamentally characterized by concrete particular structures (in some formulation of 'structure'), which can be characterized and identified in terms of the descriptions provided by fundamental physics and by using set-theoretic resources (or some other mathematical tools). Is this view immune to the difficulties just raised?

We don't think it is. There are two distinctive traits of this understanding of OSR: the emphasis on *concrete* structures, and the lack of emphasis on the individuality issue of quantum particles. If the concern with the individuality of quantum particles is dropped, and the metaphysical underdetermination between the two packages (individuals or non-individuals) is similarly dropped, then a major motivation for OSR is lost. If, however, the metaphysical underdetermination is still invoked, the objections raised above still apply. Even if the relevant structures are *concrete*, they need to be properly characterized, so that it is specified which structures one is realist about. But by invoking set-theoretic resources—or some other mathematical framework, such as category theory—in the formulation of the relevant physics, the ontic structural realist will still be committed to objects. Thus, this version of OSR doesn't overcome the difficulties that have been raised.

As another attempt to overcome those difficulties, perhaps the defender of OSR will claim that there is a metaphysical notion of structure underlying every kind of mathematical representation, something that the relevant mathematical tools simply are unable to grasp adequately. This claim, however, seems to undermine any hope of keeping the metaphysics and the epistemology properly coordinated, since it is unclear how the structures that are posited in the metaphysics could be properly characterized and known. Since the hope of adjusting the epistemology and the metaphysics is commonly found among defenders of OSR, we will examine the difficulties faced by postulating a metaphysical characterization of structure in the next section.

3 Structure and metaphysics

To examine the metaphysical nature of structure, recall that ontic structural realists countenance that science authorizes the postulation of a metaphysical entity—certain structures—about which one should be realist. And one of the motivations to go from object-oriented realism to ontic structural realism was the complaint that the former cannot determine the metaphysical nature of the entities that are posited in quantum mechanics (Ladyman 1998, p. 420, and French 2010, pp. 93 and 97, are emphatic on these points). It becomes clear then that metaphysical underdetermination is a problem for the realist. In order to address this issue, realists need to:

- determine the metaphysical nature of the entities about which they are realist;
- extract that information from science.

The first constraint is crucial in order to specify the content of the realist claim (otherwise, it is unclear what one is realist about). This is an important point: OSR is both a realist and a metaphysical view, concerned with the metaphysical nature of its posits. The second constraint is important to prevent metaphysically gratuitous additions to one's understanding of science (otherwise, it seems, object-oriented realism could not have been ruled out so easily). It should also prevent that a non-naturalist account of the nature of reality decides the issue irrespective of science. Let us now examine how well OSR fares according to these criteria.

The first question to be asked concerns the metaphysical nature of structures. Obviously, object-oriented realism was found lacking because an important scientific theory—namely, quantum mechanics—does not determine the metaphysical nature of the objects it refers to. However, can we make sure that OSR is not in the same position? *Prima facie*, it seems that OSR fares better in this respect, since it requires ontological commitment only to the common underlying structure of the two relevant metaphysical packages: one positing individuals and another positing non-individuals. And given the uniqueness of the common structure underlying these packages, it seems that there is no metaphysical underdetermination.

However, this step is not enough to characterize the metaphysical nature of that common structure. In metaphysical terms, there are still many questions that need to be answered to determine the nature of such structure (and, recall, OSR-theorists are interested in the metaphysical nature of their posits). Let us begin by recalling the discussion above of the strategy of reading "from right to left" the set-theoretic structure $\langle D, R \rangle$, that is, from relations to objects. A structure is characterized (in a loose sense) by both objects and relations, but for the structural realist only relations are primary ontologically. This is a good indication that relations are the fundamental components of the world, and indeed ontic structural realists emphasize this point (see, in particular, French 2010). But this means that in order to understand the nature of structures, we need to understand the nature of relations and of the connections they bear to objects.

Metaphysically speaking, relations are far from being uncontroversial. They are at least as controversial as properties. To speak of relations as primary components of reality, one cannot speak of them as being somehow abstracted from objects—since, in this case, they would be ontologically dependent on objects. Rather, in order to have ontological primacy, relations need to *constitute* such objects. But this still leaves open the question of what relations *are* metaphysically. There are at least two significant traditions to answer this question: relations can be thought of as universals or as particulars (in this case, as modes or tropes). Traditionally, realism about relations imply adherence to a theory of relations as universals, while nominalists are seen as adhering to tropes. However, for defenders of OSR, since they are committed to the existence of mind-independent relations, both accounts are available to characterize their metaphysical nature. A third option consists in arguing that tropes and universals can live peacefully together, with tropes being counted as instances of universals (this is the approach taken, for instance, by Lowe 2006).

How do ontic structural realists choose between these options (not to mention others that could be added to this list, since it was not meant to be exhaustive)? To avoid a "metaphysics floating free from science", one must provide an answer that is somehow endorsed by our best scientific theories—recall, once again, the fate of objects in some interpretations of non-relativist quantum mechanics and the tension this brought to object-oriented scientific realism. But interestingly, also in the case of quantum mechanics, no clear answer from the structural realist regarding the metaphysical status of relations is forthcoming. There is simply no evidence from quantum theory that decides the issue regarding the various approaches to relations. When the same situation occurred in the dispute about the metaphysical nature of objects, Ladyman (1998, p. 420) urged us to drop the commitment to objects and refrain from being realist about them. A realism that demands belief in entities whose metaphysical status is so ambiguous, he noted, is an *ersatz* form of realism. Now, if we adopt the same attitude toward relations—given that the same kind of underdetermination, of a metaphysical nature, is involved—we should abandon our belief in the primacy of relations. The demand for bringing our metaphysics closer to our epistemology seems to fail for OSR.

In response, the structural realist could insist that the situation of relations and objects in non-relativist quantum mechanics is entirely different. One cannot even begin to characterize the status of objects given the compatibility of the theory with individuals and non-individuals alike. But one can simply choose a metaphysical theory of relations and argue coherently for it in the context of non-relativist quantum mechanics. The only constraint is that the metaphysical characterization of relations should be compatible with physical systems that quantum laws and symmetries allow for. But it is not obvious that this constraint can be satisfied by all metaphysical theories of relations.

Once again, we note, the situation is somewhat more complex. Universals, particulars and tropes, as traditionally understood in metaphysics, make no specification regarding the particular physical configuration of the objects and relations involved. The notion of instantiation that is invoked in these concepts may presuppose space and time, but *no particular theory* of space and time is assumed. Whatever assumptions about space and time that are presupposed in a given formulation of non-relativist quantum mechanics can be easily incorporated into these metaphysical accounts of relations. It is, thus, unclear that the constraint to the effect that the metaphysical characterization of relations be consistent with quantum-mechanical laws and symmetries rules out any such metaphysical theories. But this means that the metaphysical nature of relations in this context is left entirely unspecified. In the end, precisely the same kind of underdetermination that the structural realist identified in the case of quantum objects is also found in the case of relations.

Perhaps structural realists could insist that these categories—universals, particulars, tropes—simply do not apply to relations (let alone to structures). It is a category mistake to ask questions of this kind about the metaphysical nature of relations. But this is clearly not right. It certainly makes sense to ask whether the relation 'being smaller than' is instantiated by two objects, or whether such relation would exist even if there were no objects that satisfy it. To deny the aptness or the intelligibility of these questions amounts to making the structural realist's notion of relation (and the corresponding notion of structure) mysterious.

As an alternative, ontic structural realists could make two moves: (i) They could take the concept of structure as primitive and articulate a new metaphysical theory that is not subject to the objections that were raised above. (ii) They could abandon the categories of ordinary metaphysics, including their connections with common sense notions, and develop a distinctive metaphysics with an entirely new understanding of relations. The resulting view need not be any more mysterious than the one that invokes ordinary categories.

However, at this stage, both suggestions are no more than promissory notes. With regard to (i), until a fully developed primitive understanding of structure is articulated, and until it is shown that such understanding overcomes the objections raised above, while still being compatible with the realist component of OSR, the very content of OSR is in question. With regard to (ii), one can, of course, simply reject the usual categories of metaphysics. But if ontic structural realists take this road, the onus is on them to show that whatever new categories they come up with are well understood and adequate to the task at hand, namely, to illuminate the nature of relations and the fundamental structures of reality. As things stand now, no such accounts of structure and relations have been developed. One would need to wait for them before any proper assessment could be made.

This situation—the metaphysical underdetermination at the level of fundamental relations—emerges from taking seriously the two requirements on realism that OSR is expected to satisfy: the metaphysical responsibilities that realists have (of specifying the content of their realism) and a naturalistic methodology (which includes how to address issues in metaphysics). That is, those requirements have now turned against OSR itself. Since both requirements cannot be satisfied in the case of OSR—we cannot under those constraints determine the true metaphysical nature of the structure of the world—it seems that something must go. Obviously, abandoning realism is the most radical option in this case. However, it seems that it is the only option left given the ontic structural realists' (justified) resistance to speculative metaphysical additions to scientific theories.

Perhaps one alternative for the defender of OSR would be to follow Maudlin (2007, Chapter 3) and accept that none of the standard accounts are correct about actual science. Indeed, Maudlin advances an alternative based on quantum field theories according to which we should investigate the metaphysics of fiber bundles, the mathematical structure used to construct such theories. Without entering into the fine details of the proposal, it seems that it would offer little comfort for OSR. To build a fiber bundle, the basis of the theory, one must begin with a base space, which in this case is space-time. Obviously, if space-time is understood substantivally, objects are re-introduced. Alternatively, if it is understood relationally, the problem of the nature of those relations strikes back, and we are back to where we started. So, even if Maudlin's proposal provided a viable alternative to traditional accounts of the nature of relations, OSR-theorists couldn't benefit from it.

Additional problems concerning the metaphysical nature of structures plague OSR.[11] An important one concerns the identity of structures. Do structures themselves, as metaphysical entities, have identity? If they do, then it seems some form

[11] To ask questions about the metaphysical nature of structures is not forbidden, since ontic structural realists admit that philosophy of science is also in the business of dealing with metaphysical issues arising from science.

of individuality should be attributed to them. If they do not, then they may be rightly called *non-individuals*. Which is the case?

This question is better answered in the context of the particular mathematical framework that is adopted.[12] If some classical set theory is used to characterize structures, there is no option but to recognize that structures have identity. Their identity results from the identity of the objects and relations that characterize the structures, given the axiom of extensionality. In other words, the identity of structures emerges from the identity of the objects that compose them, which in classical set theories are individuals (French and Krause 2006, Chapter 6, and Krause 2010). As a result, in this framework, structures are individuals. However, if an alternative set theory is adopted, a different picture emerges. To be specific, let us consider, once again, quasi-set theory. As noted above, this is a nonclassical set theory in which it is possible to study objects for which identity conditions are not well defined. In this theory, there are atoms for which identity does not apply, such as non-individuals introduced in some formulations of quantum mechanics. As a result, one can build structures satisfying a formal version of the permutation symmetry in quantum mechanics: structures that have domains with the same quantity of indiscernible elements and with the same kinds of relations are themselves indiscernible (see French and Krause 2006, p. 296, theorem 26). Thus, the resulting structures do not have identity conditions, and are properly considered non-individuals. In the end, whether structures are individuals or non-individuals depends on the particular framework that is adopted.

If this is correct, two considerations should trouble the defender of OSR. First, each mathematical framework is committed right from the start with one of the two metaphysical packages about objects mentioned above: classical set theory with the view that those entities are individuals, quasi-set theory with the view that those entities are non-individuals. Hence, to argue that one or the other framework is better equipped to characterize the relevant structures entails taking a position on the individuality versus non-individuality issue—a subject about which ontic structural realists are supposed not to take a stand, given the metaphysical underdetermination argument they invoke. Moreover, and this is the second problem, since quasi-set theory also provides the mathematical basis for a formulation of quantum mechanics that is empirically equivalent to the standard one (see Domenech et al. 2008), there is no easy way to decide between the two frameworks by considering quantum theory alone. The decision regarding which of these frameworks should be adopted rests ultimately on which conception of quantum objects is favored. But, once again, this is an issue about which OSR should not manifest itself. In the end, OSR seems unable to address properly the metaphysical issue of the individuality of structures.

This argument poses special difficulties for Saunders (2003, p. 129) and Ladyman and Ross (see 2007, p. 179, where they endorse Saunders' point). In an attempt to deny that reality has a fundamental level composed of objects, all of them conceive of

[12]We consider the mathematical framework for clarity purposes only. If a metaphysical characterization of structure is advanced independently of any such framework, problems analogous to those raised here will also emerge. After all, the issue of whether the structures in question are individuals or not can always be raised, and the problems discussed in this paragraph will return.

objects as dependent on structures while characterizing objects themselves as structures too. As we can put it, every ‘object’ is itself a structure, composed of relations and objects; the latter themselves are structures as well, and so on, with no fundamental level that is not understood in terms of structures. However, leaving aside the problem of characterizing the fundamental structure (if there is one) without invoking further objects, there is an additional difficulty. Provided that we can sensibly ask about the individuality or non-individuality of a structure (which clearly we can), it seems that we are just back to where we started. What is the nature of the structures (that play the role of objects) in quantum mechanics? Are they individuals or non-individuals? Once again, however this question is answered, as we noted, ontic structural realists face problems.[13]

It may be argued that the metaphysical problem of the identity of structures should be treated independently of any particular framework that is used to characterize structures, and thus any objection that is raised to a particular framework is only of limited value. We agree with the premise, but deny the conclusion. Questions of the individuality or not of structures can be raised as soon as any particular account of structure is advanced. Provided the account is presented explicitly and with enough detail, precisely the same issues we have raised about a particular framework can be formulated to the relevant account of structure. This issue—of the individuality or not of the resulting structures—is general enough, and can always be raised provided enough specificity is given to the structures under consideration. In this sense, the issue is not a byproduct of the particular framework, or of the particular account, structural realists adopt to articulate the notion of structure.

But perhaps talk of identity of structures is different from talk of identity of quantum particles, and thus ontic structural realists who invoke underdetermination regarding the latter need not be committed to any underdetermination regarding the former. However, we don’t see how this could be consistently sustained. After all, the central aspect of the structural realist approach to quantum particles is to conceptualize them in terms of structures. According to OSR, what these particles ultimately are is nothing more (and nothing less) than what is given by quantum mechanics. And since the theory fails to settle the issue of the ultimate nature of these particles (in particular, whether they are individuals or not), all there is to them are their structural features. This maneuver rightly moves the issue to the level of structures. But it also invites the question of whether these structures are individuals or not. As we just argued, however this question is answered, difficulties emerge. Thus, the structural realist would be hard pressed to maintain the underdetermination regarding quantum particles but reject the underdetermination regarding structures.

An additional problem regarding the metaphysical nature of structures emerges if we return to the issue of the relation between structures and the objects they give rise to. Recall that, in OSR, objects are admitted only as secondary entities, which are re-conceptualized in terms of the relations that constitute the structure. The details of

[13]We focus on the particular category of individuality (or non-individuality) of the relevant structures, rather on some other category in metaphysics, since this is the one invoked by ontic structural realists in their case for metaphysical underdetermination. So this is the relevant category to consider in this context.

this re-conceptualization are not clear in the literature: eliminativists such as French argue that objects depend ontologically on structures, while others, such as Ladyman, accept that the relations constituting the structure contextually individuate objects (for a brief account of the differences, see French and Ladyman 2011). Remember that structures are posited to accommodate scientific change: through radical theory change, the objects referred to by distinct theories may change, but some underlying common structure is preserved. We argue that this characterization of the relationship between structures and objects is problematic for structural realism.

The problem can be simply stated as follows: given that structure gives rise to objects (which are read off from the relations), how can one make sense of the disparate objects that emerge in distinct theories that share part of an underlying structure? Since some part of the structure is the same in the old and in the new theories, at least one of two options should obtain: (i) some features of the resulting objects should be the same in distinct theories, that is, there is also a form of objectual continuity through theory change, or (ii) since some structural preservation should be maintained throughout, this induces some continuity at the level of objects too, since these objects are characterized in terms of the relevant structures. However, both options entail a form of objectual continuity through theory change, something the structural realist has banned, given the pessimist meta-induction.

Note that the objection here is not that ontic structural realists are ultimately committed to distinguishing between structure and nature (see Psillos 1999). Rather, the difficulty is that their view involves continuity at the level of objects that clashes with the approach they have taken on the pessimistic meta-induction. This approach rejects any objectual continuity and proposes the corresponding elimination of objects from their ontology. Maybe ontic structural realists will insist that this degree of continuity—to the extent that it emerges from continuity at the level of structures—should be expected and embraced, and that no difficulty is, in fact, posed in this case. But we don't think this is right. As long as ontic structural realists insist that objects play no role in making sense of theory change, they are in no position to recognize any such objectual continuity—on pain of just reintroducing the objects they were trying to avoid. The result, in this case, would be an eventual commitment to standard, object-oriented realism.[14]

Perhaps the structural realist will note that the resulting continuity emerges only at the level of supervenient objects, but not at the fundamental level. It is unclear, however, how to make sense of this suggestion, given that objects emerge from whatever structures that are considered fundamental enough to be preserved in theory change. In fact, commitment to objects results directly from the way structural realists conceive of the relation between objects and structures in terms of metaphysical dependence. Let us elaborate on this point.

As suggested by French (2010), the relationship between objects and structures is one of metaphysical dependence. In the less radical version of OSR, in which objects exist but depend on relations, the proposal is formulated in this way: necessarily, the

[14]In this respect, there is a concern for ontic structural realists that their view may collapse into standard scientific realism. Psillos (1999) raises this point for the epistemic version of the view.

identity of the objects depends on the identity of the relations (French 2010, p. 105). So, in this case, it is plausible to think that, given the relations and the structures, one necessarily obtains some specific kinds of objects. On the more radical version of OSR, which eliminates objects, the dependence relation is, obviously, more radical. The essence of the objects obtained from the structure depends on the essence of the structure: it is part of the essence of the objects that they exist only if the relevant structure exists (French 2010, p. 106). In this case, even if the objects end up not existing as primary entities, the resulting entities have their essences characterized by the structure, which is something metaphysically robust. In both cases, given the relations, we have well determined objects. So, our objection—regarding the commitment to objectual continuity in OSR—is, in fact, supported by the conceptual machinery of metaphysical dependence.

One way out for structural realists would be to divide structures into two components: essential and surplus. This distinction would allow them to leave behind the features of objects that are abandoned when theories change: they are part of the surplus structure. The new theory adds to the underlying structure some additional essential structure as well as some surplus structure. The former will be preserved in the next case of theory change (in order to account for structural accumulation), while the latter accounts for the features of objects that will be abandoned in the next scientific revolution. However, this move has serious shortcomings.

First, by positing some essential structure that gets accumulated, structural realists end up admitting that in the long run (even if it is supposed to be a very long run), as scientific theories get closer to the truth, the objects will get progressively closer to being fixed by the accumulated relations, and so realism about objects will be justified too (even if only in an ideal limit). Not only would structural realists be able to know such objects, but also, after a reasonable number of revolutions, the accumulated structural content would allow them to determine the nature—the central features—of the objects. In other words, positing an essential structural component seems to entail the introduction of objects with well-defined natures. But this leads to object-oriented realism rather than structural realism.

Second, if the structural realist does not allow for some fixed, essential structure to be preserved through scientific revolutions—allowing for modifications even in the parts considered essential—then there is no reason to suppose that in the long run, after many instances of theory change, any structure will be ultimately preserved. In other words, there may be, over time, complete structural loss. (This is, of course, a version of the pessimist meta-induction for structures.) In this case, there is no reason to be realist about structures to begin with, since structures may not get preserved in theory change. In either case, realism about structures is in trouble: either objects are eventually reintroduced, or structures are lost forever.

Another possible way out would be to deny that there is such a close link between objects and relations. Relations do give rise to objects, but there is enough space for variation so that distinct theories may have the same relations and completely different objects. This line of reasoning, however, leaves the relation between structures and objects completely unspecified: relations may give rise to objects in an arbitrary way. One of the challenges for OSR is to account for the structural reconstruction of objects in actual science, to explain how the objects in scientific theories are characterized in structural terms given their actual scientific characterization. That is, there are objects even in OSR, but they must be reconstructed in structural terms. By severing the relation between structures and objects, it becomes impossible to account for the characterizations of objects that are actually provided in science, which do have well determined, non-arbitrary features. In this case, the idea of structure as a primary component of reality seems to be a source of arbitrariness, making the ontology of structures look implausible. The dependence of objects on structures should allow us to infer most of the features of objects, not to introduce them in a completely arbitrary way. So this suggestion will not work either.

4 Realism and structure

Given the considerations above, ontic structural realists are unable to specify the nature of the structure they are supposed to be realist about. There is underdetermination both at the mathematical and the metaphysical levels. Moreover, the choice between the various options cannot be made based on structural features alone, and requires the appeal to pragmatic and other non-structural factors. But this compromises the realist component of the view. Finally, realists who posit a metaphysics of structure along the lines found in OSR seem unable to maintain that science has a major role in specifying their metaphysics, since, in the end, scientific theories are unable to settle the relevant metaphysical issues about structure.

Two diagnoses can be extracted from the above arguments: either realism about structures is untenable or some other feature of OSR needs to be revised. If OSR is the best combination of realism and structuralism in philosophy of science that is also able to make sense of quantum physics, perhaps the realist component needs to be dropped. The very idea that there is a true, fundamental, underlying structure of the world—in whose existence we must believe—is difficult to make sense of, as the above arguments have indicated. So, by abandoning that idea, one can pursue freely a version of structuralism for which those problems are not a menace. One such option is, for instance, structural empiricism (see Bueno 2011). Another option consists in keeping realism but abandoning the idea that the world is only structure, embracing some form of object-oriented realism. This path is, of course, rejected by OSR, and it is, thus, a non-starter in the present discussion.

But perhaps one still wants to hold on to some form of realism and develop a metaphysics of structure. In this case, one needs to acknowledge that the truth or plausibility of the proposed metaphysics will not be settled on purely scientific grounds. By giving up on a strict naturalistic methodology in the metaphysics of science, one can introduce discussions about theoretical virtues in metaphysics, and then invoke those virtues to claim that OSR fares better than the alternatives, at least on pragmatic grounds. However, if a naturalistic metaphysics must go, then we must abandon the idea that OSR is a metaphysics tailored to fit our physics, and without this most cherished motivation, OSR is leveled with other metaphysical packages, disputing priority on *a priori* grounds. In this case, an inconvenient form of "metaphysics floating free from science" may be introduced in the dispute—an ingredient that ontic structural realists do not welcome and which leaves the realist component of their view widely open for anti-realist attacks.

Thus, the available options incur costs for the defenders of OSR. One could abandon realism or perhaps adopt a form of metaphysical optimism that the realist who is strictly scientific is unwilling to embrace. In the end, it may not be so easy to secure the best of both worlds—the price tag may be just a bit too high.[15]

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[15]Our thanks go to Newton da Costa, Steven French, Décio Krause, and James Ladyman for extremely helpful discussions about the issues examined in this work. An earlier version of the paper was presented at the Epistemology and Philosophy of Science Workshop at the University of Miami. Many thanks for all the feedback we received, in particular, from Ed Erwin, Fredrik Haraldsen, Peter Lewis, Yuki Onishi, and Harvey Siegel. Peter Lewis also gave us detailed and perceptive comments on the entire manuscript, which led to significant improvements. We are very grateful for his help. Thanks are also due to two anonymous reviewers for this journal who also provided extensive and extremely helpful comments. Needless to say, we are responsible for any mistakes and infelicities that remain.

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