Complex Systems from the Perspective of Category Theory: I. Functioning of the Adjunction Concept

Abstract

We develop a category theoretical scheme for the comprehension of the information structure associated with a complex system, in terms of families of partial or local information carriers. The scheme is based on the existence of a categorical adjunction, that provides a theoretical platform for the descriptive analysis of the complex system as a process of functorial information communication.

Keywords : Complex Systems, Information Structures, Localization Systems, Coverings, Adjunction, Sheaves.

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1 Introduction

Recently there has been a considerable interest in the foundational issues related with the modelling and comprehension of complex systems in the physical and social sciences. In this work we claim that the resolution of these issues necessitates the adoption of a simple but prevailing epistemological principle. According to this principle, the analysis of a complex system, and the consequent comprehension of its behavior, may be fruitfully performed in terms of interlocking families of simple, sufficiently understood partially or locally defined systems, which are constrained to satisfy certain appropriate compatibility relations. The simple systems may be conceived as localization devices, as information filters or as modes of perception of the complex objects, the internal structure and functioning of which, will be hopefully recovered by the interconnecting machinery governing the local objects. This point of view effectively necessitates a contextual scheme for the modelling of a complex system, as an interconnected family of simple ones interlocking in a non-trivial fashion, where the contexts are specified by the qualitative features of the simple systems. In order to explicate such a modelling scheme for complex systems, a suitable mathematical language has to be used. The language of Category theory [1-7] proves to be appropriate for the implementation of this idea in a universal way. The conceptual essence of this scheme is the development of a sheaf theoretical perspective [8-10] on the study of complex systems.

According to our knowledge, category theoretical approaches to the study of systems have been considered in great detail, from a different modelling perspective, in [11-15]. Insightful philosophical and semantical aspects related with the use of category theoretical frameworks of reasoning, from various viewpoints, are discussed extensively in references [1,5,6,16-20].

2 Philosophy of the Scheme

Category theory provides a general theoretical framework for dealing with systems formalized through appropriate mathematical structures putting the emphasis on their mutual relations and transformations. The basic categorical principles that we adopt in the subsequent analysis are summarized as follows:

[i] To each kind of mathematical structure used to model a system, there corresponds a **category** whose objects have that structure, and whose morphisms preserve it.

[ii] To any natural construction on structures of one kind, yielding structures of another kind, there corresponds a **functor** from the category of the first specified kind to the category of the second. The implementation of this

principle is associated with the fact that a construction is not merely a function from objects of one kind to objects of another kind, but must preserve the essential relationships among objects.

According to the aforementioned principles and the general philosophy of category theory, it is reasonable to assume that a complex system can be understood by means of appropriately specified maps having as their domains intentionally depicted structures modelling the behavior of simple, sufficiently understood systems, and codomains, an operationally or theoretically specified structure arising from the behavior of the complex system. In the great majority of the cases, any concrete map from a simple or local domain object proves to be not adequate for a complete determination of the totality of information contained in the complex system, and hence, it captures only a limited amount of information associated with it. Evidently, it includes the amount of information related to a specified context, or mode of perception, or a localization environment, and thus, it is inevitably constrained to represent the abstractions associated with the intentional aspect of its use. This theoretical problem may be tackled, only by the simultaneous employment of a sufficient number of structure preserving maps from the well comprehended relatively simple or local objects to the complex object of enquiry.

This process is formalized categorically by the concept of a covering system, where the specified maps play the role of covers of the complex object. In more detail, the notion of local is characterized by using a topology (in the general case a Grothendieck topology on a category [3,8,9,10]), the axioms of which express closure conditions on the collection of covers. In this sense the information available about each map of the specified covering system may be used to determine the complex object itself. In this paper we will avoid to mention Grothendieck topologies on categories explicitly in order to avoid unnecessary technical complications in the exposition of the arguments.

The notion of a covering system must be necessarily accompanied by the establishment of a suitable notion of compatibility between the various covers of the complex object. This is necessary since it guarantees an efficient pasting code between different local viewpoints on the complex object.

The efficiency of the pasting code is formalized in category theory language by the concept of sheaf, which expresses essentially gluing conditions, namely the way by which local data can be collated into global ones. It provides the appropriate vehicle for the formalization of the relations between covering systems and properties, and, furthermore, provides the means for studying the global consequences of locally defined properties in any attempt of probing the structure of a complex system.

Essentially a map which assigns a set to each object of a topology is called a sheaf if the map is defined locally, or else the value of the map on an object can be uniquely obtained from its values on any cover of that object. Categorically speaking, besides mapping each object to a set, a sheaf maps each covering map in the topology to a restriction function in the opposite direction. We stress the point that the transition from locally defined properties to global consequences happens via a compatible family of elements over a covering system of the complex object. In this perspective a covering system on a complex object can be viewed as providing a decomposition of that object into simpler objects. The sheaf assigns a set to each element of the cover, or else each intentionally specified piece of the complex object. A choice of elements from these sets, one for each piece, forms a compatible family if the choice respects the mappings by the restriction functions and if the elements chosen agree whenever two pieces of the cover overlap. If such a locally compatible choice induces a unique choice for the object being covered, a global choice, then the condition for being a sheaf is satisfied. We note that in general, there will be more locally defined or partial choices than globally defined ones, since not all partial choices need be extendible to global ones, but a compatible family of partial choices uniquely extends to a global one.

The above general scheme accomplishes the task of comprehending entirely the complex object through covering families of well known local objects pasted together appropriately, in case there exists an isomorphism between the operationally or theoretically specified structure representing a complex system and the sheaf of compatible local viewpoints imposed upon it.

3 Categories of the Universe of Discourse

We describe a system by means of a category $\mathcal{K}$, according to principle [i] of the proposed categorical scheme. This category is required to be small [2,3,8,9], by construction, such that, the families of its objects and morphisms form genuine sets. Its objects, K, are structures used to describe the behavior of a system, characterized in every concrete case by means of operational or theoretical means. Usually these structures correspond to event or observable algebras associated with a system. In other cases these structures may specify topological or spatial features of a system, or even provide a description of its behavior in terms of logic. We will adopt a homogenous treatment of all the possible structural characterizations corresponding to the objects of $\mathcal{K}$, referring to them as information structures. We wish to make clear that the word information is conceived in its broadest possible meaning, and it is used for reasons of homogeneity in the exposition of the ideas. The arrows in the category of information structures associated with a system are required to be structure preserving maps. This is a reasonable requirement, since it is desirable to have a preservation of the specified information structure, in each concrete case, by maps to or from objects of the same category. The same requirement may also be conceived as an implication of an ontological principle, rooted in the philosophy of categories, according to which, in order to understand a structure it is necessary to understand the morphisms preserving it.

We may assume the distinction between simple and complex systems. A simple system, according to the above general description, admits a representation in terms of a category of information structures whose qualitative features are well understood. In this sense, a system is characterized as complex relatively to the complexity of its information structures with respect to the ones used to model the behavior of simple systems.

By taking into account the previous distinction, we describe a complex system by means of a small category $\mathcal{Z}$. Its objects (called complex objects), Z, are complex information structures, whereas its arrows are structure preserving maps between them. We claim that a complex system can be comprehended in terms of a functorial construction, realized for each complex object Z in $\mathcal{Z}$, as an interlocking family of incoming maps from the domains of intentionally depicted structures, that characterize the behavior of simple, sufficiently understood systems. In this perspective we construct a category $\mathcal{Y}$, whose objects, Y, are intentionally selected algebraic or topological structures, called partial or local information carriers, whereas its arrows are structure preserving maps of these carriers. Their role is inextricably connected with the philosophy of being attached to a complex object as localization devices, or information filters or even as modes of perception. The epistemological purpose of their introduction is, eventually, the construction of a covering system of a complex object, signifying an intentional structured decomposition of an information structure in terms of partial or local carriers, such that the functioning of the former, will be hopefully approximated, or completely recovered, by the interconnecting machinery governing the organization of the covering system. Evidently, each local or partial information carrier, includes the amount of information related to a filtering process, objectified by a specified context, or a localization environment, and thus, it represents the abstractions associated with the intentional aspect of its use.

A further claim, necessary for the development of the proposed scheme, has to do with the technical requirement that the category of information structures has to meet a condition, phrased in category theoretical language, as cocompleteness [2,3]. This condition means that the category of information structures has arbitrary small colimits. The existence of colimits expresses the basic intuition that a complex object may be conceived as arising from the structured interconnection of partially or locally defined information carriers in a specified covering system.

4 Functorial Modelling

After the specification of the categories of the universe of discourse according to principle [i] of the categorical philosophy, it is necessary to relate them by means of a functorial collective environment as an implementation of principle [ii]. Specification of functorial relations is crucial for, both, the comprehension of a complex system in terms of structured information contained in the organization of a family consisting of partially or locally defined information carriers, and equally significant, for the qualification of this family as a covering system of the information structure, associated categorically, with the complex system itself.

4.1 Functor of Local Coefficients

We define a functor of local or partial coefficients, or equivalently a shaping functor for a complex information structure, $\mathbf{A} : \mathcal{Y} \to \mathcal{Z}$, which assigns to information carriers in $\mathcal{Y}$, constituting the category of shapes or models or viewpoints, the corresponding information structures from $\mathcal{Z}$, and to Y-structure preserving morphisms the corresponding Z-structure preserving morphisms. The functor of local coefficients may be considered as a functor that shapes an information structure by forgetting, from the perspective of Z, any simplifying characterization associated with a filtering process objectified by its carrier.

4.2 Category of [Information Carriers]-Variable Sets

At a first stage we assume that the abstract quantification of the information gathered by each filtering process in the domain of an intentionally specified qualitative context, gives rise to a set, which represents in the environment of the category of sets $\mathcal{S}$, the elements of the information content associated with each particular partial or local information carrier. We mention parenthetically, that, addition and multiplication over $\mathbf{R}$ induces the structure of a ring or of an $\mathbf{R}$-algebra on each specified set. At a second stage, we wish to express the intuitively simple idea that the family of all sets of the kind specified by the qualitative features of the information carriers, may be organized together in a suitable category, expressing exactly the variation of the information content over the carriers, as well as the structural preservation of the information engulfed in them. This idea can be formalized by the construction of the functor category of presheaves over the category of partial or local information carriers, conceived as a kind of varying information set. If we consider that $\mathbf{Sets}^{\mathcal{Y}^{op}}$ is the universe of partial or local information carriers structures modelled in **Sets**, and $\mathcal{Z}$ that of complex information structures, then the functorial nature of the first category is suited to represent the varying world of localization filters of information, associated with intentionally depicted abstraction mechanisms of decomposition of a complex system. We mention parenthetically that the functor category of presheaves can be qualified, at a later stage, as a category of sheaves only after the specification of an appropriate pasting code that guarantees compatibility among different local or partial information filters on each complex object.

Most remarkably, the functor category of presheaves on partial or local information carriers $\mathbf{Sets}^{\mathcal{Y}^{op}}$, provides an exemplary case of a category known as topos [8,9,10,15]. A topos can be conceived as a well defined notion of a set varying over a specified base domain. Furthermore, it provides a natural example of a many-valued truth structure, which remarkably is not ad hoc, but reflects genuine constraints of the surrounding universe.

We proceed by a detailed description of the functor category of presheaves as follows: For the category of partial or local information carriers $\mathcal{Y}$ we will be considering the category $\mathbf{Sets}^{\mathcal{Y}^{op}}$ of all contravariant functors from $\mathcal{Y}$ to $\mathcal{S}$ and all natural transformations between these. A functor $\mathbf{P}$ is a structure-preserving morphism of these categories, that is it preserves composition and identities. A functor in the category $\mathbf{Sets}^{\mathcal{Y}^{op}}$ can be thought of as constructing an image of $\mathcal{Y}$ in **Sets** contravariantly, or as a contravariant translation of the qualitative language of $\mathcal{Y}$ into that of **Sets**. Given another such translation (contravariant functor) $\mathbf{Q}$ of $\mathcal{Y}$ into $\mathcal{S}$ we need to compare them. This can be done by giving, for each object Y in $\mathcal{Y}$ a transformation $\tau_Y : \mathbf{P}(Y) \longrightarrow \mathbf{Q}(Y)$ which compares the two images of the information carrier Y in the environment of $\mathcal{S}$. Not any morphism will do, however, as we would like the construction to be parametric in Y, rather than ad hoc. Since Y is an object in $\mathcal{Y}$ while $\mathbf{P}(Y)$ is in $\mathcal{S}$ we cannot link them by a morphism. Rather the goal is that the transformation should respect the information carriers structure preserving morphisms of $\mathcal{Y}$, or in other words, the interpretations of $v : Y \longrightarrow C$ by $\mathbf{P}$ and $\mathbf{Q}$ should be compatible with the transformation under τ. Then τ is a natural transformation in the functor category $\mathbf{Sets}^{\mathcal{Y}^{op}}$.

It is useful to think of an object $\mathbf{P}$ of $\mathbf{Sets}^{\mathcal{Y}^{op}}$ as a right action of $\mathcal{Y}$ on a set which is partitioned into kinds parameterized by the information carriers objects in $\mathcal{Y}$, and such that, whenever $v : C \longrightarrow Y$ is a structure preserving morphism between information carriers, and p is an element of $\mathbf{P}$ of information kind Y, then pv is specified as an element of $\mathbf{P}$ of kind C, such that the following conditions are satisfied

$$p1_Y = p, \quad p(vw) = (pv)w, \quad vw : D \longrightarrow C \longrightarrow Y$$

Such an action $\mathbf{P}$ is equivalent to the specification of a set varying over the category of partial or local information carriers, or briefly, $\mathcal{Y}$-set. The fact that any morphism $\tau : \mathbf{P} \longrightarrow \mathbf{Q}$ in the category $\mathbf{Sets}^{\mathcal{Y}^{op}}$ is a natural transformation is expressed by the condition

$$\tau(p, v) = \tau(p)(v)$$

where the first action of v is the one given by $\mathbf{P}$ and the second by $\mathbf{Q}$.

Of paramount importance for the coherence of functorial modelling in the category of presheaves $\mathbf{Sets}^{\mathcal{Y}^{op}}$ is the existence of the embedding functor $\mathbf{y}_{\mathcal{Y}} : \mathcal{Y} \longrightarrow \mathbf{Sets}^{\mathcal{Y}^{op}}$. The embedding functor associates to each information carrier A of $\mathcal{Y}$ the $\mathcal{Y}$-set $\mathbf{y}_{\mathcal{Y}}(A) = Hom_{\mathcal{Y}}(-, A) := \mathcal{Y}(-, A)$, whose Y-th kind is the set $\mathcal{Y}(Y, A)$ of $\mathcal{Y}$ morphisms $Y \longrightarrow A$, with action by composition: $xv : C \longrightarrow Y \longrightarrow A$. This is a functor because for any structure preserving morphism between information carriers $A \longrightarrow D$, there is obtained a $\mathcal{Y}$-morphism $\mathcal{Y}(-, A) \longrightarrow \mathcal{Y}(-, D)$, that exhibits a functorial behavior under composition $A \longrightarrow D \longrightarrow E$, due to the associativity of composition in $\mathcal{Y}$. In view of the functorial embedding $\mathbf{y}_{\mathcal{Y}} : \mathcal{Y} \longrightarrow \mathbf{Sets}^{\mathcal{Y}^{op}}$, the partial or local information carrier A, may be thought of as the representable object $\mathbf{y}_{\mathcal{Y}}(A)$ in $\mathbf{Sets}^{\mathcal{Y}^{op}}$, determined completely, by all structure preserving morphisms from the other information carriers in $\mathcal{Y}$. At a further stage of development of the same philosophy, for any $\mathcal{Y}$-set and for any information carrier A of $\mathcal{Y}$, the set of elements of $\mathbf{P}$ of kind A is identified naturally with the set of $\mathbf{Sets}^{\mathcal{Y}^{op}}$-morphisms from $\mathbf{y}_{\mathcal{Y}}(A) \longrightarrow \mathbf{P}$. This observation, in effect, permits the consideration of the elements of $\mathbf{P}$ of information carrier kind A, as morphisms $\mathbf{y}_{\mathcal{Y}}(A) \longrightarrow \mathbf{P}$ in $\mathbf{Sets}^{\mathcal{Y}^{op}}$.

4.3 Category of Elements of an [Information Carriers]-Variable Set

Since, by construction $\mathcal{Y}$ is a small category, there is a set consisting of all the elements of all the sets $\mathbf{P}(\mathrm{Y})$, and similarly there is a set consisting of all the functions $\mathbf{P}(f)$. We will formalize these observations about the specification of $\mathbf{P} : \mathcal{Y}^{op} \longrightarrow \mathbf{Sets}$ by taking the disjoint union of all the sets of the form $\mathbf{P}(\mathrm{Y})$ for all information carriers Y of $\mathcal{Y}$. The elements of this disjoint union can be represented as pairs (Y, p) for all objects Y of $\mathcal{Y}$ and elements $p \in \mathbf{P}(\mathrm{Y})$. We can say that we construct the disjoint union of sets by labelling the elements. Now we may construct a category whose set of objects is the disjoint union just mentioned. This structure is called the category of elements of $\mathbf{P}$, denoted by $\mathbf{G}(\mathbf{P}, \mathcal{Y})$. Its objects are all pairs (Y, p), and its morphisms $(\acute{Y}, \acute{p}) \longrightarrow (Y, p)$ are those morphisms $u : \acute{Y} \longrightarrow Y$ of $\mathcal{Y}$ for which $pu = \acute{p}$. Projection on the second coordinate of $\mathbf{G}(\mathbf{P}, \mathcal{Y})$, defines a functor $\mathbf{G_P} : \mathbf{G}(\mathbf{P}, \mathcal{Y}) \longrightarrow \mathcal{Y}$. $\mathbf{G}(\mathbf{P}, \mathcal{Y})$ together with the projection functor $\mathbf{G_P}$ is called the split discrete fibration induced by $\mathbf{P}$, and $\mathcal{Y}$ is the base category of the fibration. The word discrete refers to the fact that the fibers are categories in which the only arrows are identity arrows. If Y

is an information carrier object of $\mathcal{Y}$, the inverse image under $\mathbf{G_P}$ of Y is simply the set $\mathbf{P}$(Y), although its elements are written as pairs so as to form a disjoint union. The construction of the fibration induced by $\mathbf{P}$, providing the category of elements of an [information carriers]-variable set, is an application of the categorical Grothendieck construction [10].

5 Functorial Information Exchange

5.1 Adjunction between Presheaves of Local Carriers and Information Structures

The notion of adjunction [21], provides the conceptual ground concerning the comprehension of complex systems in terms of structured families of partial or local structures of information carriers, according to the guiding epistemological principle of the modelling scheme, and is based on the categorical construction of colimits over the category of elements of an [information-carriers]-variable set $\mathbf{P}$.

For this purpose, we consider the category of complex objects $\mathcal{Z}$, the shaping functor $\mathbf{A}$, and subsequently, we define the functor $\mathbf{R}$ from $\mathcal{Z}$ to presheaves given by

$$\mathbf{R}(Z) : Y \mapsto Hom_{\mathcal{Z}}(\mathbf{A}(Y), Z)$$

We notice that the set of objects of $\mathbf{G}(\mathbf{R}(Z), \mathcal{Y})$ consists of all the elements of all the sets $\mathbf{R}(Z)(Y)$, and more concretely, has been constructed from the disjoint union of all the sets of the above form, by labelling the elements. The elements of this disjoint union are represented as pairs $(Y, \psi_Y : \mathbf{A}(Y) \longrightarrow Z)$ for all objects Y of $\mathcal{Y}$ and elements $\psi_Y \in \mathbf{R}(Z)(Y)$. Taking into account the projection functor, defined previously, this set is actually a fibered structure. Each fiber is a set defined over a partial or local information carrier.

A natural transformation τ between the presheaves on the category of information carriers $\mathbf{P}$ and $\mathbf{R}(Z)$, $\tau : \mathbf{P} \longrightarrow \mathbf{R}(Z)$ is a family τ_Y, indexed by information carriers Y of $\mathcal{Y}$, for which each τ_Y is a map

$$\tau_Y : \mathbf{P}(Y) \rightarrow Hom_{\mathcal{Z}}(\mathbf{A}(Y), Z)$$

of sets, such that the diagram of sets below, commutes for each structure preserving morphism $u : \acute{Y} \rightarrow Y$ of $\mathcal{Y}$.

$$\begin{array}{ccc}
\mathbf{P}(Y) & \xrightarrow{\tau_Y} & Hom_{\mathcal{Z}}(\mathbf{A}(Y), Z) \\
\mathbf{P}(u) \downarrow & & \downarrow {}^{*}\mathbf{A}(u) \\
\mathbf{P}(\acute{Y}) & \xrightarrow{\tau_Y} & Hom_{\mathcal{Z}}(\mathbf{A}(\acute{Y}), Z)
\end{array}$$

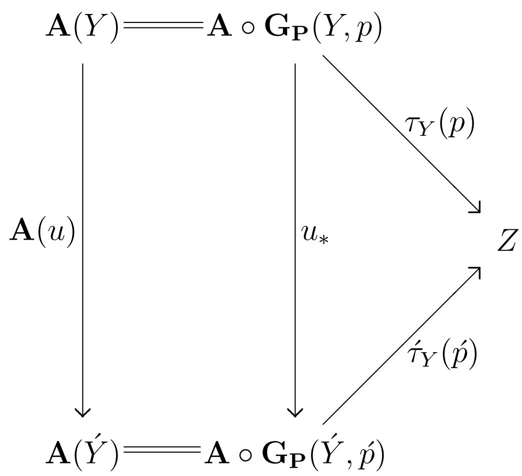
Adopting the perspective of the category of elements of the [information carriers]-variable set P, the map τ_Y, defined above, is identical with the map:

$$\tau_Y : (Y, p) \rightarrow Hom_{\mathcal{Z}}(\mathbf{A} \circ G_{\mathbf{P}}(Y, p), Z)$$

In turn, such a τ, can be conceived as a family of arrows of $\mathcal{Z}$, indexed by objects (Y, p) of the category of elements of the presheaf $\mathbf{P}$, namely

$$\{\tau_Y(p) : \mathbf{A}(Y) \to Z\}_{(Y,p)}$$

Thus, from the viewpoint the category of elements of $\mathbf{P}$, the condition of the commutativity of the diagram above, is translated into the condition that for each arrow u the following diagram commutes:



This diagram clearly shows that the arrows $\tau_Y(p)$ form a cocone from the functor $\mathbf{A} \circ G_{\mathbf{P}}$ to an information structure Z. Moreover, by taking into account, the categorical definition of the colimit, we conclude that each such cocone emerges by the composition of the colimiting cocone with a unique arrow from the colimit $\mathbf{LP}$ to the complex object Z. Put differently, there is a bijection which is natural in $\mathbf{P}$ and Z

$$Nat(\mathbf{P}, \mathbf{R}(Z)) \cong Hom_{\mathcal{Z}}(\mathbf{LP}, Z)$$

From the above bijection we are driven to the conclusion that the functor $\mathbf{R}$ from $\mathcal{Z}$ to presheaves, given by

$$\mathbf{R}(Z) : Y \mapsto Hom_{\mathcal{Z}}(\mathbf{A}(Y), Z)$$

has a left adjoint $\mathbf{L} : \mathbf{Sets}^{\mathcal{Y}^{op}} \to \mathcal{Z}$, which is defined for each presheaf of partial or local information carriers, $\mathbf{P}$ in $\mathbf{Sets}^{\mathcal{Y}^{op}}$, as the colimit

$$\mathbf{L}(\mathbf{P}) = Colim\{\mathbf{G}(\mathbf{P}, \mathcal{Y}) \xrightarrow{\mathbf{G_P}} \mathcal{Y} \xrightarrow{\mathbf{A}} \mathcal{Z}\}$$

Consequently there is a pair of adjoint functors $\mathbf{L} \dashv \mathbf{R}$ as follows:

$$\mathbf{L} : \mathbf{Sets}^{\mathcal{Y}^{op}} \overset{\longleftarrow}{\longrightarrow} \mathcal{Z} : \mathbf{R}$$

Thus, we have constructed an adjunction which consists of the functors $\mathbf{L}$ and $\mathbf{R}$, called left and right adjoints with respect to each other respectively,

$$\begin{array}{ccc} Nat(\mathbf{P}, \mathbf{R}(Z) & \xrightarrow{\mathbf{r}} & Hom_{\mathcal{Z}}(\mathbf{LP}, Z) \\ \| & & \| \\ Nat(\mathbf{P}, \mathbf{R}(Z) & \xleftarrow{\mathbf{l}} & Hom_{\mathcal{Z}}(\mathbf{LP}, Z) \end{array}$$

as well as, the natural bijection: $Nat(\mathbf{P}, \mathbf{R}(Z)) \cong Hom_{\mathcal{Z}}(\mathbf{LP}, Z)$.

As an application, we may consider the bijection defining the fundamental adjunction for the representable presheaf of the category of partial or local information carries $\mathbf{y}[Y]$.

$$Nat(\mathbf{y}[Y], \mathbf{R}(Z)) \cong Hom_{\mathcal{Z}}(\mathbf{Ly}[Y], Z)$$

We note that when $\mathbf{P} = \mathbf{y}[Y]$ is representable, then the corresponding category of elements $\mathbf{G}(\mathbf{y}[\mathbf{Y}], \mathcal{Y})$ has a terminal object, namely the element $1 : Y \longrightarrow Y$ of $\mathbf{y}[Y](Y)$. Therefore the colimit of the composite $\mathbf{A} \circ \mathbf{G}_{\mathbf{y}[Y]}$ is going to be just the value of $\mathbf{A} \circ \mathbf{G}_{\mathbf{y}[Y]}$ on the terminal object. Thus we have

$$\mathbf{Ly}[Y](Y) \cong \mathbf{A} \circ \mathbf{G}_{\mathbf{y}[Y]}(Y, 1_Y) = \mathbf{A}(Y)$$

Hence we characterize $\mathbf{A}(Y)$ as the colimit of the representable presheaf on the category of information carriers.

We conclude that the following diagram commutes:

$$\begin{array}{ccc} \mathcal{Y} & & \\ \mathbf{y} \downarrow & \searrow^{\mathbf{A}} & \\ \mathbf{S}ets^{\mathcal{Y}^{op}} & \xrightarrow{\mathbf{L}} & \mathcal{Z} \end{array}$$

A technically and conceptually important further step, refers to the categorical equivalent presentation of the colimit in the category of elements of the functor $\mathbf{P}$ as a coequalizer of coproduct as follows:

$$\coprod_{v:\acute{Y} \to Y} \mathbf{A}(\acute{Y}) \underset{\eta}{\overset{\zeta}{\rightrightarrows}} \coprod_{(Y,p)} \mathbf{A}(Y) \xrightarrow{\chi} \mathbf{P} \otimes_{\mathcal{Y}} \mathbf{A}$$

where, $\mathbf{P} \otimes_{\mathcal{Y}} \mathbf{A} = \mathbf{L}_A(P)$. In the diagram above the second coproduct is over all the objects (Y, p) with $p \in \mathbf{P}(Y)$ of the category of elements, while the first coproduct is over all the maps $v : (\acute{Y}, \acute{p}) \longrightarrow (Y, p)$ of that category, so that $v : \acute{Y} \longrightarrow Y$ and the condition $pv = \acute{p}$ is satisfied.

This presentation is significant for the purposes of the present scheme, because it reveals the fact that the left adjoint functor of the adjunction is like the tensor product $- \otimes_{\mathcal{Y}} \mathbf{A}$. In order to illustrate the analogy observed, we simply take $\mathcal{Z} = \mathbf{Sets}$. Then the coproduct $\amalg_p \mathbf{A}(Y)$ is a coproduct of sets, which is equivalent to the product $\mathbf{P}(Y) \times \mathbf{A}(Y)$ for $Y \in \mathcal{Y}$. The coequalizer is thus the definition of the tensor product $\mathcal{P} \otimes \mathcal{A}$ of the set valued factors:

$$\mathbf{P} : \mathcal{Y}^{op} \longrightarrow \mathbf{Sets}, \qquad \mathbf{A} : \mathcal{Y} \longrightarrow \mathbf{Sets}$$

$$\amalg_{Y, \acute{Y}} \mathbf{P}(Y) \times Hom(\acute{Y}, Y) \times \mathbf{A}(\acute{Y}) \underset{\eta}{\overset{\zeta}{\rightrightarrows}} \amalg_Y \mathbf{P}(Y) \times \mathbf{A}(Y) \xrightarrow{\chi} \mathbf{P} \otimes_{\mathcal{Y}} \mathbf{A}$$

According to the above diagram, for elements $p \in \mathbf{P}(Y)$, $v : \acute{Y} \to Y$ and $\acute{q} \in \mathbf{A}(\acute{Y})$ the following equations hold:

$$\zeta(p, v, \acute{q}) = (pv, \acute{q}), \qquad \eta(p, v, \acute{q}) = (p, v\acute{q})$$

symmetric in $\mathbf{P}$ and $\mathbf{A}$. Hence the elements of the set $\mathbf{P} \otimes_{\mathcal{Y}} \mathbf{A}$ are all of the form $\chi(p, q)$. This element can be written as

$$\chi(p, q) = p \otimes q, \quad p \in \mathbf{P}(Y), q \in \mathbf{A}(Y)$$

Thus if we take into account the definitions of ζ and η above, we obtain

$$pv \otimes \acute{q} = p \otimes v\acute{q}, \quad p \in \mathbf{P}(Y), \acute{q} \in \mathbf{A}(\acute{Y}), v : \acute{Y} \longrightarrow Y$$

We conclude that the set $\mathbf{P} \otimes_{\mathcal{Y}} \mathbf{A}$ is actually the quotient of the set $\amalg_Y \mathbf{P}(Y) \times \mathbf{A}(Y)$ by the equivalence relation generated by the above equations. It is easily proved that the presentation of the colimit as a tensor product can be generalized for $\mathcal{Z}$, being any cocomplete category, as required in the specification of the category of information structures, representing the behavior of a complex system.

5.2 Interpretation of the Adjunction

The existence of the categorical adjunction explained above, provides a theoretical platform for the formulation of a scheme of comprehending a complex system, by viewing its decomposition in terms of partial or local information carriers, as a process of functorial information communication. If we consider, as in 4.2, that $\mathbf{Sets}^{\mathcal{Y}^{op}}$ is the universe of [information-carriers] variable sets, and $\mathcal{Z}$ that of complex information structures, then the functor $\mathbf{L} : \mathbf{Sets}^{\mathcal{Y}^{op}} \longrightarrow \mathcal{Z}$ can be understood as a translational code from partial or local information filters to the information structure describing a complex system, whereas the functor $\mathbf{R} : \mathcal{Z} \longrightarrow \mathbf{Sets}^{\mathcal{Y}^{op}}$ is a translational code in the inverse direction. In general, the content of the information is not possible to remain completely invariant translating from one language to another and back, in any information exchange mechanism. However, there remain two ways for an [information-carriers] variable set $\mathbf{P}$, characterized as a multiple levels information window, to communicate a message to an information structure Z. Either the information is exchanged in the terms of the complex object, specified by the information structure Z, to be analyzed, with $\mathbf{P}$ translating, which can be represented as the structure preserving morphism $\mathbf{LP} \longrightarrow Z$, or the information is exchanged in the terms of the information carriers, with Z translating, that, in turn, can be represented as the natural transformation $\mathbf{P} \longrightarrow \mathbf{R}(Z)$.

In the first case, from the perspective of Z information is being communicated in the complex object's terms, while in the second, from the perspective of the structured information window, $\mathbf{P}$, information is being communicated in the partial or local descriptive terms of the category of carriers. The natural bijection then corresponds to the assertion that these two distinct ways of communicating are equivalent. Thus, the philosophical meaning of the adjunction, signifies an amphidromous dependence of the involved, information descriptive, languages in communication, that assumes existence at the level of relating relations. This process is realized operationally in any methodology of extraction of the information content enfolded in a complex system's information structure, through the pattern recognition features of intentionally specified localization environments or modes of perception. In turn, this process gives rise to a variation of the information collected in the partial information carriers filtering systems, for probing the information structure associated with a complex system, which is not always compatible. In Part II, we will specify the necessary and sufficient conditions for a full and faithful representation of the informational content included in a complex information structure in terms of information carriers localization systems, being qualified as covering systems of the information structure of a complex system. At the present stage we may observe that the representation of a complex information structure as a categorical colimit, resulting from the same adjunction, reveals an entity that can admit a multitude of instantiations, represented by different shaping functor coefficients in partial or local information filtering carriers.

Acknowledgments: The author is member of the EDGE Research Training Network HPRN-CT-2000-00101, supported by the European Human Potential Programme.

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