Récoltes et Semailles, Part I

The life of a mathematician

Reflections and Bearing Witness

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By way of a Preface ..... written January 30th, 1986

Only the preface remains before sending Recoltes et Semailles to the printer. And I confess that I had every intention of writing something appropriate . Something quite reasonable, for once. No more than 3 or 4 pages, yet well expressed, as a way of opening this enormous tome of over a thousand pages. Something that would hook the skeptical reader, that would make him willing to see what there was to be found in these thousand pages. Who knows, there may well be things here which might concern him personally!

Yet that's not exactly my style, to hook people. But I was willing to make an exception in this case, just this once! It was essential if this book was to find an "editor crazy enough to undertake the venture" , (of publishing this clearly unpublishable monster).

Oh well, it's not my way. But I did my best. And not in a single afternoon, as I'd originally intended. Tomorrow it will be three weeks I've been at work on it, that I've watched the pages pile up. What has emerged can't in any sense be called a "preface" . Well, I've failed again. At my age I can't make myself over: I'm not made to be bought and sold! Even if I had every intention of being pleasing, to others and to myself ...

What I've ended up with is a kind of long "promenade" with commentary, through my life's work as a mathematician. A promenade for the lay public - for those who "don't understand a thing about mathematics". And for myself as well, because I've never embarked on such a stroll. I saw myself engaged in uncovering and in talking about things that have always been more or less proscribed. Coincidentally they are also those matters which

I consider most basic to my work and my opus. These things have nothing to do with mathematical technicalities. You are the judge of my success in this enterprise, which I agree is really a bit insane. I will be satisfied if I have made you feel something of what I have felt, things which most of my colleagues don't know how to feel. It may be that they have become to erudite, or received too many honors. Such things cause one to lose contact with the essentials.

In the course of this "Promenade through an opus" I will also be talking about my own life. And, here and there, of the purpose behind Recoltes et Semailles. Following the "Promenade" you will find a letter, (dated May of the previous year) . This letter was to be sent to my former students and my "old friends" in the world of mathematics. This also, is not technical in any sense. It should be readable by anyone who has an interest in learning, via a living document, about all of those odds and ends which have culminated in the production of Recoltes et Semailles. Even more than the Promenade, the Letter should give you an idea of a certain kind of atmosphere, that of the mathematical world in its largest sense. And, also, (in the Promenade) , you may find my manner of expression a bit unusual, as you may find the mentality that naturally employs such a style - one that is far from being understood by the rest of the world

In the Promenade, and here and there in Récoltes et Semailles I will be speaking of the nature of mathematical work. It is work that I understand very well from first hand experience. Most of what I say will apply equally well, I think, to all creative labor, and all activities of discovery. It will apply at least for what is known as 'intellectual' work, which is done mostly 'in one's head', and to writing. Work of this sort is distinguished by the hatching out and by the blossoming of our understanding of certain things which we are interrogating.

To take an example in the other direction, passionate love is, also, driven by the quest for discovery. It provides us with a certain kind of understanding known as 'carnal' which also restores itself, blossoms forth and grows in depth. These two impulses -that which animates the mathematician at his desk (let's say), and that which impels the lover towards the loved one - are much more closely linked than is commonly believed, or, let us say, people are inclined to want to believe. It is my wish that these pages of Récoltes et Semailles will make its reader aware of this connection, in his own work and in his daily life.

Most of the time In the course of this excursion we will be concerned with mathematics itself, properly speaking. I will be saying almost nothing about the context in which this work takes place, or of the motivations of individuals which lie outside the work itself. This runs the risk of giving me, or the mathematician or scientist in general, a somewhat flattering image, and for that reason distorted- the sort of thing one sees in speaking of the "grand passion" of the scientist, without restrictions. That is to say, something along the lines of the grandiose "Myth of Science" (with a capital S if you please!); the heroic "myth of Prometheus" which writers have so often indulged in (and continue to do so) , for better or worse. Only the historians, and then not always, have been able to resist the seductions of this myth. The truth of the matter is that it is universally the case that, in the real motives of the scientist, of which he himself is often unaware in his work, vanity and ambition will play as large a role as they do in all other professions. The forms that these assume can be in turn subtle or grotesque, depending on the individual. Nor do I exempt myself. Anyone who reads this testimonial will have to agree with me .

It is also the case that the most totally consuming ambition is powerless to make or to demonstrate the simplest mathematical discovery - even as it is powerless (for example) to "score" (in the vulgar sense) . Whether one is male or female, that which allows one to 'score' is not ambition, the desire to shine, to exhibit one's prowess, sexual in this case. Quite the contrary!

What brings success in this case is the acute perception of the presence of something strong, very real and at the same time very delicate. Perhaps one can call it "beauty", in its thousand-fold aspects. That someone is ambitious doesn't mean that one cannot also feel the presence of beauty in them; but it is not the attribute of ambition which evokes this feeling....

The first man to discover and master fire was just like you and me. He was neither a hero nor a demi-god. Once again like you and me he had experienced the sting of anguish, and applied the poultice of vanity to anaesthetize that sting. But, at the moment at which he first "knew" fire he had neither fear nor vanity. That is the truth at the heart of all heroic myth. The myth itself becomes insipid, nothing but a drug, when it is used to conceal the true nature of things.

I intend in Récoltes et Semailles to speak of both aspects : of the passion for knowledge, and the passion of fear and the antidotes of vanity used to curb it. I make the claim that I understand , or at least am well acquainted with, the passion for knowledge. (Yet perhaps one day I will discover, to my amazement, to what extent I've been deceiving myself) . Yet when it comes to fear and vanity, and the insidious ways in which these block creativity, I am well aware that I have not gotten to the root of this enigma. Nor do I know if I will ever see through to the end of this myself in the years remaining to me.

Over the course of writing Récoltes et Semailles there emerged two images, representing two fundamental aspects of the human adventure: These are the child (alias the worker), and the boss . In the Promenade on which we are about to embark, we will be dealing almost exclusively with the child. He also figures in the section entitled "The child and the Mother". The meaning of this term will, I trust, become clear as we proceed.

Yet in the remainder of the work it is the boss who will be at center-stage: he isn't the boss for nothing! To be more precise, one isn't talking about a single boss, but of various bosses of different enterprises being maintained concurrently. At the same time, these bosses have a way of resembling one another in their essential nature.

Once one begins to talk about bosses, there have to be villains. In Part I of the section entitled "Complacency and Restoration" (Fatuité et Renouvellement), which comes right after the introductory material (Prelude in 4 Movements), it is I, above all, who am the "villain"! In the remaining 3 sections, its the others. Everyone gets a turn!

In other words one can expect to find, along with a number of more or less profound philosophical reflections and some 'confessions'(without contrition), several "acid sketches" (portraits au vitriol- to use the expression of one of my colleagues who has found himself somewhat mistreated), as well as a host of vigorous "operations" which have not been sanitized. Robert Jaulin* has assured me (only partly joking) that what I'm doing in Récoltes et Semailles is a kind of 'ethnography of the mathematics community' (or perhaps the sociology)

(*) Robert Jaulin is an old friend of mine. It is my understanding that he, vis-a-vis the community of ethnologists, is regarded, (as I am in the mathematics establishment) as something of a 'black sheep'.

It is extremely flattering of course to learn, (without having been aware of it), that one is engaged in real scholarship! It is a fact that in the inquiry portion of this enterprise I have watched (at some risk to myself) the passage across these pages of a considerable portion of the mathematics establishment, as well as friends and colleagues of less exalted status.That has come back to me over the last few months during which I've started sending out a provisional first edition of Récoltes et Semailles.

Unquestionably my testimony has had the effect of tossing a brick through a glass window! Echoes of every sort (save that of boredom) have resonated from everywhere. Frankly this was not what I'd expected. And there's been lots of silence too, the kind that speaks volumes

Clearly I still have a lot to learn, about all the things going on in the private retreats of others, such as ex-students and those former colleagues who seem to be doing pretty well for themselves (my apologies ?I meant to say in the 'sociology of the mathematics community'!) To all those who, in their own way, have contributed to this 'sociological research' with which I occupy my elderly days, I of course express my profoundest gratitude.

Needless to say I have been most receptive to the enthusiastic responses. There have also been those colleagues , rare enough, who have shared with me their feelings and their experiences about the state of crisis, and the extreme degradation, which lies at the heart of the contemporary mathematics community, of which they are members.

Among those who, outside of this circle, have been among the first to give this testimony a warm reception I wish to single out Sylvie and Catherine Chevalley(*), Robert Jaulin, Stéphane Deligeorge, Christian Bourgeois.

(*)Sylvie and Catherine Chevalley are the widow and daughter of Claude Chevalley, the colleague and friend to whom I've dedicated the central core of Récoltes et Semailles (R&SIII,"The Key to the Yin and the Yang"). I will be speaking of him in many places, and of his role in my personal journey.

Well, here we go! Let us begin our Promenade through the work of a lifetime,as a way also of speaking of that life itself. It will be a long voyage indeed, of more than a thousand pages, each of them filled with substance. It has required the whole of my life to make this voyage, which is far from finished, and more than a year to reconstruct it, page upon page. The words have sometimes come hesitantly, as they attempt to give the full import of my experience, the understanding of which has also come hesitantly ?like the ripe grape buried in the winepress that may offer resistence to the force applied to crush it.Yet even at those times when it appeared that words were pouring out, tumbling over one another in their urge for release, they were not being strewn at random on the page. Each of them has been carefully weighed, either at the moment of their emergence or in subsequent consideration, and appropriately modified if too light or too heavy.

Thus, don't expect that this reflection-witnessing-voyage will make for facile reading, in a day or even in a month. It is not intended for the reader who wishes to come to the end of it as quickly as possible. One can't really speak of "endings", much less "conclusions" in a work like Récoltes et Semailles , no more than one finds such things in my life or in yours.

Think of it like a wine fermented in the depths of someone's being for a lifetime. The last glass will be neither better nore worse than the first, or the hundredth. They are all alike, and they are all completely different. And if the first goblet is spoiled, the whole vat from which it comes is likewise spoiled. Far better to drink good water than bad wine!

Yet, when one finds a good wine, it is best to sip it slowly, and not when one is one the run.

Introduction

It must have been in July of 1984 that I had an extraordinary dream. When I use the expression "extraordinary", I refer only to the impression that it made on me afterwards. The dream itself appeared as the most natural thing in the world, without any sort of fanfare Ð to such an extent that, even after awakening I attached no importance to it, and buried it somewhere in the secret dungeon of the unconscious so as to get on to the business at hand.

Since the previous day I'd been reflecting on my relationship to mathematics, indeed it was probably the first time ever that I'd thought to consider this matter seriously. Indeed if I was doing so now, it was only because I was forced into it. So many strange, even violent things had happened over recent months and years, one might call them veritable explosions of the passion for mathematics that continue to erupt without showing any sign of diminishing, that I simply could not proceed further without taking an overview of what had been going on.

The dream I'm referring to had no scenario, no specific acts or activities. It contained but a single frozen image, one that was at the same time remarkably alive. It was a human head seen in profile, scanned from left to right. The head was of a mature man, beardless, with wild head wrapped around its brow like a bright powerful halo. The strongest impression made by this head was of a joyous, youthful vitality, which seemed to spring directly from the supple and vigorous arching of its neck (sensed more than seen). The facial expression was more that of a mischievous delinquent than of a responsible or settled adult, thrilled by the recollection of some trick he'd gotten away with or was about to do. It gave off an intense love of life, playful, content with itself.

Nobody else was present, no-one to play the role of observer, and "I" to look at or contemplate this being, of whom one saw only the head. Yet the perception of this head, or let us say of the atmosphere which it evoked, was extremely intense. Nor was anyone else present to record impressions, comment on them, or give a name to the person being observed, to call him "this or that". There was only this intensely vital object, the man's head, and an awareness of that vitality.

When I awoke and reviewed the various dreams which had passed that night, the one with the man's head did not seem of any particular significance, there was nothing in it that might make me cry out: you ought to be looking at me! Reviewing this dream in the quiet comfort of my bed, I was driven by the natural desire to put a name to this apparition. Nor did I have far to search: once the question was posed it was more than obvious that the head I'd seen in my dream was none other than my own.

It's not a bad thing, I told myself, it takes some doing to see one's own head in a dream as if it were that of another! The dream gave the impression of having arisen by accident, much as when one finds a 4-leaf clover, (or even one of 5 leaves), yet aroused no other reaction, so that shortly afterwards I felt free to go on my way as if nothing had happened.

That's more or less the way it happened. Happily, as often occurs in situations of this sort, as a way of putting my conscience to rest I wrote it down, which was enough to set off an meditation that would hook up with that which I'd begun the previous evening. Then, little by little, the thoughts of that day began to involve me more and more with the event of that dream, its single coherent image, and the message it carried for myself.

This is not the proper place to expand on all that this day's meditation taught and delivered Ð to be more precise, that which the dream taught and delivered once I'd put myself in a receptive state of mind which enable me to gather what it was trying to tell me. I can say however that the first fruit of that dream, and of this state of receptivity, was a spontaneous upwelling of fresh energy. It was this energy which would give me over the following months, the stamina to persist in the long meditation that, by means of patient and persistent work, resulted in overcoming many internal resistances and opinions.

Its been five years since I began to pay attention to a certain number of my dreams. This was the first "messenger dream" which didn't have the usual characteristics of such a dream, which generally combines impressive scenery and an heightened intensity of vision which can at times be overwhelming. This was far more subtle, low key, with nothing in it to force one to focus one's attention, the very model of discretion- one could, it appeared, take it or leave it. A few weeks later I received a messenger dream of the conventional sort, running the gamut of drama, even of savagery, which had the effect of bringing to a sudden end a long period of frantic mathematical activity. The only connection between the two dreams was that neither in the one nor in the other was there any kind of observer. Like the parabolic trajectory of a stone under the effect of gravity, this dream was showing me something that was going on in my life, quite apart from any influence or even knowledge of it on my part Ð indeed, things which I'd gone out of my way to deny. It was this dream, above all, which impressed upon me the urgency for a labor of internal reflection. Beginning a few weeks later, this labor continued for another six months. I will have occasion to speak of it in the latter part of the section entitled "Recoltes et Semailles" which opens the present volume and gives its name to the entire opus. (*)

(*)Note in particular section 43 : The Boss as Kill-Joy -or the Pressure Cooker

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The reason that I've opened this introduction with the evocation of this dream, this image-vision of myself (Traumgesicht meiner selbst , as I labeled it in my notes German) is because over the past few weeks the recollection of this dream has persistently returned to me.

At the same time the long meditation on the "past of the life of a mathematician" has been drawing to its close. In retrospect, speaking honestly, the 3 years that have gone by since the appearance of this dream have been years of crystallization and maturation as I've moved towards the enunciation of a message that is both simple and lucid.

That dream showed me to myself, "as I am" . Equally, it is clear that the being in my waking life is not at all the same as the one that the dream revealed to me. Obstacles and restrictions going back a long distance have obstructed (and continue to obstruct) the possibility for me to be simply myself. Over these years, even though the recollection of this dream came to me only rarely, it must certainly have acted on me in several ways. One should not think of it as a kind of model or ideal which I felt myself obliged to imitate, but merely of a discrete reminder of a kind of joyous simplicity which at one point "was me", which reveals itself under numerous guises, which seeks to be liberated from that which weighs upon it, and which continues to blossom forth. This dream was the tie, both delicate and vigorous, between a present still laden down with past burdens, and an immanent "tommorow" which takes the form of a seed, a "tommorow" which is in my present, and which has certainly always been within me. Certainly if in those weeks this rarely recollected dream had been present within me , it had to be the level of thinking that is weighed and analyzed. I should have realized that the work that I was engaged on and bringing to a conclusion, constituted a new step in the direction of the message about myself which it conveyed.

This is, at the present moment, my sense of the meaning of Récoltes et Semailles, and of the intensive work that had occupied the last two months. Only now, after it has been finished, have I fully realized the importance of what I was doing. In the course of this effort I've known many moments of happiness, a happiness that has often been mischievous, comic and exuberant. And there have also been moments of sadness, those moments when I had to relive all the frustrations and sorrows which have been my lot over recent years. But I have not known a single moment of bitterness. I end this work with the total satisfaction of one who has brought a difficult work to completion. I have evaded nothing, not matter how minor, which in my heart I knew had to be said, so I am not left with the dissatisfaction of having left something undone.

It has always been clear to me while I was writing this testimonial that it could not please everybody. In fact, it's quite possible that I've found a way to disappoint everyone. However my intention has only been to cast an overview on the simple details, the basic events of my past (and present also) as a mathematician, to convince myself (better late than never), without reservation or the shadow of a doubt, of what they were and what they are; and, once having taken this road, to describe what I've learned as simply as possible.

1. The introspective effort which has led to the creation of Récoltes et Semailles started out as an introduction to the first volume (still in the process of being completed) of "Defining Stacks"(*)

((*)" Pursuing stacks = A la poursuite des champs" Author(s): Grothendieck, A. (Alexandre) Publication: Publication: [S.l. : s.n., Year: 1983 Description: vi, 12, 593 ; 30 cm.Note(s): Typewritten manuscript)

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the first mathematical work I've prepared for publication since 1970. I'd written the first few pages at a critical moment, in June of last year, and I resumed my reflections less than two months ago, at exactly the place where I'd left off. It became clear to me that there were a great many things that needed to be looked at and spoken of, but thought that they could be disposed of by a somewhat dense introduction of 30 or 40 pages. During the following months, up to this very moment when I am engaged in writing up a new introduction to what had been intended as an introduction, I believed at the start of each day that it would be finished on that day, or on the next, or without a doubt the day after.

When at the end of several weeks I'd saw that it was approaching the first 100 pages, that introduction was promoted to the status of "introductory chapter"! Another two weeks went by, by which time the dimensions of this "chapter" had grown beyond those of all other chapters in the work being prepared (all, with the exception of the final one , complete at the time of writing these lines) , I finally understood that its proper place was not in a book about mathematics, in which this testimonial would be quite inappropriate. It deserved a volume all by itself, which I intended to be Volume I of those "Reflections on Mathematics" which I intend to develop in the years to come, as sequel to "Defining Stacks ".

I do not want to claim that Récoltes et Semailles, (which is the first volume in the series of "Mathematical Reflections" (and which will be followed by two or three volumes of "Defining Stacks" , to begin with),)ought to be considered as a kind of "introduction" to the Reflections . I rather see this first volume as the foundation of all that is to come, or , to express it better, something to set the tone, the spirit in which I intend to undertake this new voyage, that which I intend to pursue over the years to come, which will lead me I know not where.

To round out these comments about the subject of the major part of the present volume, I will give several indications of a practical nature. The reader should not be surprised to find in the text of Récoltes et Semailles that there are references to the "present volume" - by which is meant the first volume (History of Models) of "Defining Stacks", of which I am still in the process of writing the introduction. I have not bothered to "correct" these passages because I don't want to lose the spontaneity with which the text was written, nor the authenticity of its evidence, not only on a distant past, but of the moment at which it is being written.

It is for this same reason that my revisions of the first draft of the text have been restricted to obvious clumsiness in the style, or of a way of stating things that might be so confusing as to seriously compromise one's understanding of what is being stated. These revisions have sometimes led me to a more subtle or clearer grasp of the matter. Modifications of greater weight, to nuance, render precise, complete or sometimes to correct what is stated in the text, have been put into over fifty numbered footnotes, grouped together at the end of this treatise, and which constitute more than 25% of the text. (*)

(May 28th) This refers only to the text of the first part of Récoltes et Semailles, "Complacency and Restoration". At the time I wrote these lines, the second part hadn't yet been written.

These are indicated by signs like (1), etc. Among these notes one can identify about twenty or so, which are at the same level of importance (either in terms of their length or their substance), as the fifty or so "sections" or "paragraphs" into which the reflection as a whole has naturally organized itself. These lengthy notes have been included in the table of contents.

As one might expect, to some of these very long footnotes have been added several sub-notes. These are then included at the end of the text of the original footnote, with the same sort of cross-reference, save for very short notes which can be found on the same page, with cross-references such as (*) or (**) .

It was a source of delight for me to adjoin names to each section of this text, as well as to some of the more substantial notes. Indeed it was indispensable that I do so in order for me to resituate myself in the text. It goes without saying that these names were invented afterwards, given that when I began a section or a footnote I had no idea where my thoughts would lead. It was the same de fortiorti with respect to names (such as "Work and Discovery", etc.) by which are designated the 8 subdivisions , I through VIII, in which, as a afterthought, the fifty sections of text were organized.

With regards to the content of these 8 subdivisions I will limit myself to a few very brief observations. The first two parts , I (Work and Discovery) , and II (The Dream and the Dreamer) , contain the elements of a meditation on the nature of mathematical work, and on the nature of the adventure of discovery in general. My own person is engaged in a fashion far more haphazard and indirect than in the parts that follow. These are the ones which, above all, have the quality of documentation and of meditation.

Parts III to VI are above all are a backward look and a reflection on my past as a mathematician , "in the world of mathematics", between 1948 to 1970. The motivation inspiring this meditation has been the desire to understand that past, as part of my effort to understand and assimilate a present of which certain elements may appear deceptive or perplexing.

Parts VII (The child at play), and VIII (The solitary adventure) are mostly concerned with the evolution of my relationship to mathematics between 1970 up to the present, that is to say, from the time I left the "world of mathematics" for good. Herein I've examined the motives, forces and circumstances that have led me (to my great surprise) to once again pick up a public role in mathematics (by writing and publishing Mathematical Reflections), after an interruption of over 13 years.

2. I feel obliged to say a few words on the subject of the two other texts which compose, along with Récoltes et Semailles, the present volume with the same name. The "Outline for a Programme" (Esquisse d'un Programme) sketches the principal themes of the mathematical reflection which I've been engaged upon over the last 10 years. I hope to develop several more of these in the coming years, in the form of a series of casual reflections about which I've already spoken , the "Mathematical Reflections" . This sketch reproduces the text of a report which I wrote last January, to support my application for a research post with the CNRS. (Centre Nationale de Recherche Scientifique) . Its been included in the present volume because, as one can readily see, the ambition of this programme surpasses my limited personal resources, even if I were given the chance to live another hundred years, and because I intend to make a selection from among its themes that will be followed up to the best of my ability.

The "Thematic Outline" ("Esquisse thematique des principaux travaux mathematiques de A. Grothendieck")was written in 1972 in defense of another candidacy, (for a professorship with the College de France) . It outlines, in a thematic catalogue, those things which I believe to be my principal contributions to mathematics. This text reflects my tendencies at the time it was written, at a time when my interest in mathematics was extremely marginal, to say the least. Thus this outline is little more than a dry and methodical enumeration (which, happily, made no attempt to be exhaustive.)

It is completely lacking in vision or passion, as if the things I was talking about, (through some notion of acquitting my conscience (which indeed describes what I tended to feel)) had never been animated by a living vision, nor by any passion to bring them into the light of day when they were still behind their curtains and shadow and fog.

Despite this I've decided to include this demoralized document because, I fear, to silence (imagining that it would be possible to do so) certain highly placed colleagues of a certain sort who, since my departure from the world of mathematics, choose to be somewhat disdainful of what they've amiably dismissed as my grothendieckeries .

Apparently this has become a synonym for wasting precious time on things beneath the concern of a serious and mature mathematician. Possibly this "indigestible digest" will impress them as something to be taken more seriously! As for the works I've produced under the force of passion and vision, it is only to be expected that those persons whom the world supports and acclaims, are insensitive to the things which have enchanted me. If I have written for others besides myself, it has been for those who have not stingily guarded their time and their persons, who have steadfastly persevered in examining those self-evident things which others dare not look at, who rejoice in the inimitable beauty of each of those things I've discovered, things distinguished for their intrinsic beauty from whatever came before.

If I were to attempt to place the three texts of the present volume in relation to one another, (and the role of each in the voyage first undertaken with the Mathematical Reflections), I would say that the reflection/testimonial "Ré et Semailles" , reflects and describes the spirit in which I undertook this voyage, and which gave it meaning. The "Outline of a Programme" details my sources of inspiration, which set a direction, if not exactly a destination, for this voyage into the unknown, a bit like the needle of a compass, or a vigorous Ariadne's Thread. Finally the "Thematic Outline" is a rapid run-through of a certain amount of old baggage, acquired in my past as a mathematician up to 1970, a part of which at least may be serviceable to this or that stage of my voyage (such as for example my reflections on Cohomological and Topos Algebras, which are indispensable for me in the conception of "Defining Stacks").

The order of these 3 texts, as well as their respective lengths, are an accurate reflection (with any deliberate intention on my part) of the relative importance I've assigned to them in the journey, of which the first stage is now approaching its end.

3. It remains for me to say a few words of a particular nature concerning the current voyage, initiated a bit more than a year ago, that is to say , the "Mathematical Reflections". These are convered in explicit detail in the first 8 sections of Récoltes et Semailles (i.e. in Parts I and II of the overview), when I talk about the spirit in which this voyage was undertaken, which I believe should be apparent right from the beginning of the present first volume, as it is in the volume that follows it (The History of Models, which is Volume I of "Defining Stacks"), and which is now in the process of being finished. It is therefore not necessary to go into this matter in the introduction.

   For a certainty I am unable to predict how this voyage will turn out, for it is something which is to be discovered in the course of my pursuing it. Right now I have no grand itinerary, and I don't think there will be one. As was stated before, the major themes which I'm pretty certain will serve as the inspiration for this reflection are essentially sketched in the "Outline for a Programme", which is the 'orienting text' ("texte-boussole). Among these themes is included the major theme of "Defining Stacks" that is to say the "stack" itself, which I hope to cover in the next year, in two or perhaps three volumes. With respect to this theme I wrote the following in the "Outline": "...it is something like a debt that I have contracted against myself vis-a-vis my scientific past, over a 15-year period (between 1955 and 1970). The constant refrain of this period had been the laying of the foundations of Algebraic Geometry through the development of the tools of co-homology.

   It is this, in the list of all the themes I expect to touch on, which has the strongest connection with my scientific past. And it is this which I've most regretted over the last 15 years, as the most flagrant omission among all those which I abandoned by leaving the mathematical scene, which none of my students or former friends have taken the trouble to correct. For more details on this work in progress, the interested reader may turn to the relevant section in the "Outline", or to the introduction (the 'real' one , this time!) of the first volume of "Defining Stacks".

Another legacy of my scientific past that is particularly close to me, is of course the concept of the "motive", waiting its turn to come out of the night into which it has been enshrouded since it first made its appearance over a good 15 years ago. It is not impossible that I may try to complete my work on the foundations of this subject, if no-one in a better position than I am, (younger perhaps, with fresh tools and knowledge) sees it worth his while to undertake such a project in the coming years.

I would like to use this occasion to point out that the fate of the notion of the "motive", and of several others which I brought into the light of day, those that are the most powerful and fertile, will be the subject for a discussion of about 20 pages, forming the longest footnote (and one of the last) in Récoltes et Semailles (*)

This double footnote (#'s 46 and 47), and its subnotes have been included in the second part of volume 4 of Récoltes et Semailles, "The Burial".

This footnote was afterwards subdivided into two parts :My orphans andRejection of a heritage - or the cost of a contradiction, and supplemented by 3 additional subnotes(*)

(*)These are notes 48,49,50: the subnote '48' was added later

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The totality of these 5 notes in sequence is the only place in Récoltes et Semailles where mathematical concepts are treated directly rather than being alluded to. These concepts then serve as the basis for illustrating certain contradictions at the heart of the world of mathematics, which reflect contradictions within the people themselves who make up that world.

At one point I'd thought of separating this gigantic footnote from the text from which it is derived, and placing it in the "Thematic Outline". This would have had the further advantage of putting it in proper perspective, and to inject a bit of life into a text that may appear to be something of a dry catalogue. I decided not to do this out of a wish to maintain the integrity of a testimonial for which this "mega-footnote", whether or not it pleases me, forms an integral part.

With regard to what is stated in Récoltes et Semailles regarding my intentions when I began the "Mathematical Reflections", I need only add one more thing: It suffices to quote what has already been written in a footnote (The snobbery of youth - or the defenders of purity) : "Through my whole life my ambition as a mathematician, or rather my passion and joy, has ever been to uncover self-evident truths. This is also my sole ambition in this present work. (Defining Stacks) . It remains my unique ambition for this new voyage which, a year later, is being pursued in the "Mathematical Reflections". And it is this same in these Récoltes et Semailles which (for my readers at least, if there are any) open this voyage. I would like to conclude this introduction with a few words on the pair of dedications made at the beginning of Récoltes et Semailles. The dedication " to those former students of mine to whom I've given the best of myself- and also the worst", had been decided upon since at least the previous summer, particularly when I was writing the first four sections of what was still envisaged only as an introduction to a work in mathematics. I knew very well, in other words, and in fact had known it for several years already, that that "worst part of myself" would have to be looked into - and that it was now or never! (Although I'm certain that this 'worst part' would led to another opus of a minimum of 200 pages.)

On the other hand, the dedication "to my elders", only came to me in the course of the writing, just like the name of this whole reflection (which has also become that of one of its volumes). I came to realize the great importance of their role in my life as a mathematician, a role whose effects are still very apparent today. This will no doubt become clear in the pages that follow, so there is no need to elaborate on this subject. These "elders" are, in the approximate order in which they entered my life from the age of 20 onwards: Henri Cartan, Claude Chevalley, André Weil, Jean-Pierre Serre, Laurent Schwartz, Jean Dieudonné , Roger Godement, and Jean

Delsarte. Those persons encountering me for the first time may not be aware of the degree of enthusiasm and good-will with which all of them welcomed me, or the extent to which so many of them have continued in the solid friendship and affection.

Here is the place also to mention Jean Leray, whose warm reception at the time of my first encounters with the world of mathematics (1948/49) was also an important source of encouragement. This reflection has made me realize my debt of gratitude towards each of these men, " from another world and with another destiny". This debt is in no sense a burden. To uncover it has been a source of joy, and lightened my spirit. (The end of March, 1984)

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4. (May 4...June) An unexpected development has reopened a reflection which I'd believed complete. It has initiated a veritable cascade of discoveries, large and small, over the course of the past weeks, revealing bit-by-bit a situation which is more fluid than I imagined it. In particular it has led me to enter profoundly into the details of events and situations which I'd previously referred to only in passing or by allusion. All of a sudden the "retrospective of a dozen or more pages" about the fate of a life's work, which I'd spoken of previously (Introduction, section 4) has grown to unexpected dimensions, amounting to a supplement of about another 200 pages

. By the force of circumstances and through the inner logic of this retrospective, I have been led to have to implicate others besides myself. The person in question is, more than all others myself excepted, a man bound to me with ties of affection for close to 20 years. Of him I have written (in the form of a euphemism(*) , that he had "taken the part of a student" in the early years of this close friendship based on a common passion.

On the special meaning of this "euphemism" see the footnote "The outsider",#67

For a long time in my private reflections I looked upon him as a kind of "legitimate heir" of all that I imagined I'd brought to mathematics over and above the fragments that have been published.

There must be many who already know of whom I'm speaking :Pierre Deligne

I do not apologize for making public, in these writings, a personal reflection on a personal relationship, nor to implicate someone without consulting him beforehand.It seems important that me that a situation which, for all too long, has been shrouded in confusion and obscurity, be brought into the light of day for examination. This being said, I am writing a testimonial, one that is certainly subjective and which does not claim to exhaustively treat of so delicate a situation, nor to be free of errors. Its principal merit, (like that of my earlier publications, or those on which I'm working presently), is that it exists to be consulted by those persons to whom it will be of interest. My concern is neither to convince, nor to shield myself from the possibility of being mistaken in documentation only those things which one can assume are taken for granted. My concern is only to be faithful to the truth as I see it, in order to get to the bottom of certain things and render them more understandable.

The title " The Funeral Rites" has been given to all of the footnotes relating to " The Weight of the Past". It's suitability came to me with insistent force in the course of writing this reflection (*)

(*)Near the end of this work another name also suggested itself, expressing a different, equally striking aspect of a certain picture of things that revealed itself to me in progressive stages over a period of 5 weeks. Its the title of a fable, to which I will return in its proper place: The Robe of the Emperor of China

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In it I play the role of a man whose death is anticipated, in the lugubrious company of several mathematicians (much younger than I), whose work has all been done after my "departure" in 1970, who therefore lacked the advantages of direct contact with me and my advice, who knew about my work through my writings, those published or available by other means. At that time I was already being treated as something of a corpse, to the degree that for a very long time that even the notion that people ought to meet me was not current, and that a continuous relationship (as much personal as mathematical), had unravelled about a year before.

That did not however prevent Mebkhout, going against the grain of a somewhat tyrannical and disdainful attitude of his peers (who were my students) , and in nearly total isolation, to create original and profound work through making a unexpected synthesis of my ideas with those of the school of Sato. His work opened a new insight into the cohomology of analytic and algebraic varieties; it carries the promise of a vigorous renaissance in our understanding of this cohomology. It is certain that this resurgence of research in this subject would have been carried through by now, or even several years ago, had Mebkhout found himself surrounded by the appropriate warmth and unreserved support from his colleagues, which he had formerly received from me. As it is, since October 1980 his works and ideas have supplied the inspiration and the technical means for spectacular advances in the theory of the cohomology of algebraic varieties, which has finally emerged (putting aside the results of Pierre Deligne on the Weil conjectures), after a long period of stagnation.

Incredible as it may sound, over the last four years his ideas and results have been used by "everybody" (just as mine have been), even as his name is ruthlessly ignored and suppressed, even by those who know about his work through direct association with him or who have used it as an essentially component of their own research. I know of no other period in the history of mathematics which has been guilty of such disgraceful conduct, namely, that some of the most prestigious and influential members of the community set the example for all others, to violate the most fundamental ethical principles of the mathematical calling.

I distinguish four men, all brilliant mathematicians, who have shared with me the honor to be the victims of this burial through silence and disdain . I can see how the stigma of contempt has poisoned, in each of them, the beautiful passion for mathematics that once inspired them.

Quite distinct from them I see above all two men, each of them a monument set up in places of public mathematical honor, who are now acting in official capacities in the funereal rituals, and who, at the same time, (in a more secretive sense) are burying themselves with their own hands. One of these has already been named. The other is also one of my former friends and students : Jean-Louis Verdier . Apart from occasional and brief meetings at professional conferencesm, We did not keep up contact after my "departure" of 1970. No doubt this explains why he does not figure overly much in this reflection apart from certain of his professional activities. Furthermore the motivation behind these activities, vis-a-vis his relationship to me, are not looked into at all because I don't know anything about them

If there is one inquiry which has, with great urgency, driven me over the long years, which served as a deep incentive for writing Récoltes et Semailles, which has accompanied me throughout the length of its writing, it is that of my own responsibility in the advent of a certain kind of spirit and a certain kind of tradition which have made possible the kinds of demeaning behavior to which I've alluded, from a world to which I belonged and to which I entirely identified over the more than twenty years that I was a mathematician. This self-examination has led to the discovery of distinctively fatuous attitudes in me, which manifested themselves by a disdain for colleagues less gifted than I was, and by a spirit of accomodation to mathematicians at my level. I therefore am no stranger to the kinds of attitudes which I see everywhere around me today, among persons I've loved, and among those to whom I taught a subject that I loved. One can say that it is among those whom I've badly loved and badly instructed who are setting the tone (let us not call it the law), in a world that was so dear to me, and which I finally left.

I sense an atmosphere of self-congratulation, of cynicism and contempt. "The wind bloweth where it listeth..." ... I have understood that we are reaping the whirlwind of all those blind and callous seeds which I helped to plant. And if this whirlwind has fallen back onto me, and on those whom I've left in other hands, and on those for whom I still have affection and who have had the courage to admit that I inspired them, it is not more than a tit-for-tat of which I have no-one to blame but myself, and which has much to teach me.

7. In the table of contents, under the designation of "The Funeral Rites" , I have therefore collected the impressive lost of the principal "notes" relating to that seemingly innocuous section " The Weight of the Past". These notes give meaning to the name which impressed itself upon me for this final section of the first draft of Récoltes et Semailles.

In this long processional of footnotes of multiple parentage, including those which have been added over the last 4 weeks (notes (51) to (97) are the only ones which have dates (from April 19th to May 24th(**)

(*) Note #104 of May 12 1984 has also be added. The notes from #98 onwards (with the exception of the footnote that precedes #104), form the "third wind" of this reflection, which began on September 22, 1984. All of them are dated . In a list of footnotes which had been written up on the same day, only the first is dated. Others among them which aren't dated are #44' to #50 (these form the funeral processions I, II, III) . Footnotes #'s 46, 47, and 50 are from March 30th or 31st, footnotes #44', 48, 48', 48'', 49 from the first half of April. Finally, note #44" is dated May 10th.

(**) From time to time I've weakly inverted the chronological order, to the benefit of the "logical order", whenever I felt that it was important to maintain the sense of a progression advance in my reflections. The only exceptions are the 11 footnotes (whose numeration is always preceded by the exclamation sign (!)). They refer to other footnotes which risked swelling to prohibitive dimensions. They've been placed directly after the footnotes to which they correspond. (with the sole exception of note #98, which actually relates to note #47.)

(***) When the numeral of footnote (such as (46)) is referred to within a note in the section entitled " The Weight of the Past", then the numeral of the note containing that reference (such as (50) for example) is placed immediately after the numeral of the referenced footnotes . In this example that would be #46 (50) .

(****)The numbering of a footnote which is the direct continuation of the preceding one, is always preceded by an asterisk * in the table of contents. Thus for example, *47.46 indicates that note #47 is the immediate continuation of the subject matter of #46 (which may well be different from the note that physically follows #46, which is this case is the footnote #46.9.Finally I have indicated, in the table of contents, the numbers of all the footnotes that are not the continuations of others, those which represent a "new departure" in my thinking, and have no clear place with regard to the previous reflection

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In order to lend a bit of structure to the totally of The Funeral Rites, and so that one doesn't lose one's place in the multitudes of footnotes, it seemed a good idea to me, depending on the circumstances, to adjoin at certain places a number of very suggestive subtitles. They precede and orient a long succession of consecutive footnotes united around a common theme.

I've thereby has the signal satisfaction of witnessing the gradual coming together, piece by piece, of the 10 phalanxes of the long processional assembled for my burial rites (*)

(*) September 29th. In fact there are actually 12 phalanxes, including that of the hearse (X) , and "The corpse (who is however not yet dead) " (XI) , which arrive in extremis to insinuate themselves into the processional.

Some of them modest, others imposing, some contrite, others secretly exulting, as it must always be on such occasions. One by one these advance

1. The Posthumous Pupil (whom all consider it their duty never to acknowledge)
2. The Orphans (freshly exhumed for this solemn occasion
3. The Illustrious Men in Fashion (I deserved that one)
4. The "Motives" (the latest born and most recently resurrected of all my orphans)
5. My friend Pierre D ... (the humble leader of the largest of all the phalanxes) , followed close behind by
6. The Unanimous Concord of Silent Melodies
7. The 'Colloquium" (alias 'The Perverse') , which includes a full house. For the posthumous pupil, otherwise known as the 'unknown pupil' , subsidiary funeral marches carrying flowers and crowns have been set aside.

   Finally, bringing up the rear of this imposing brigade, we watch the advancement of

8. The Pupil (hardly posthumous though even less known), alias The Boss, followed by
9. The harried troupe of all my students (forced to carry shovels and buckets) . Finally:
10. The Hearse, (holding in its keep four beautiful coffins of solid oak, their lids well screwed in place, plus the Gravedigger)

Altogether 10 phalanxes all coming together (it was high time) to move, ever so slowly, to The Ceremony.

The final nail in the coffin will be the Funeral Oration, served up with the finest of touches by none other than my old friend Pierre in person, presiding over the rites at the request and satisfaction of all present. The Ceremony terminates with a (final and definitive; at least one hopes)De Profundis, chanted as a sincere act of contrition by none other than the much lamented corpse himself, who, to the stupefaction of all present has somehow survived his own obsequies and even participated in them with the greatest imaginable satisfaction - this satisfaction providing the final note and the final Tierce de Picardie(Note: it is the translator who finds this technical term from music particularly apt.) of so memorable an entombment.

8. In the course of this final stage (so we hope) of this retrospective it was felt that an Appendix was needed for volume 1 of the "Mathematical Reflections", containing two other documents of a mathematical character, in addition to the three already alluded to (*)

(*) In addition I've considered adding a commentary to the "Thematic Outline", giving some more details about my contributions to the "themes" which are summarily passed over, as well as on the subject of the influences at work in the genesis of the major and most powerful ideas in my mathematical opus. An overview of the last six weeks has already led me to realize (much to my surprise!), to what extent Jean-Pierre Serre played the role of a "detonator" to the eruption of most of these ideas, as he did for several of the "grand projects" which I envisaged between 1955 and 1970.

Finally, there is another mathematical text (in the modern sense), the only one to figure in the body of Récoltes et Semailles, that I want to call attention to, which is the sub-footnote #87 adjoined to the footnote "The Massacre" (#87)in which I've described, with great care, my conjecture of a discrete 'variant' of the familiar Riemann-Roch-Grothendieck theorem for the continuous context. This conjecture figures (along with several others) in the treatment at the close of the seminar SGA 5 of 1965/66, of which, (along with other work) not a trace is to be found anywhere in the volume that was published 11 years later under the title "SGA 5" . The vicissitudes of this critical seminar at the hands of some of my students, and their ties to a certain Operation SGA 4 1/2 are unfolded bit by bit in the investigation carried out in the footnotes #63''', 67, 67', 68, 68', 84,85, 85', 86, 87, and 88.

Another mathematical discussion, about the possibility of putting together a topos (to the extent possible) for the known cases in which there exists a formal duality of the sort I've called the "6 operations", is to be found in the subnote #81.2 of the footnote "Risk free insurance to a thesis advanced on credit", #81.

The first text reproduces and comments on a report in two parts which I did between 1968 and 1969 on the works of Pierre Deligne (much of which remain unpublished to this day) , corresponding to the mathematical activity at the Institute des Hautes Etudes Scientifiques during the years 1965/67/68.

The other text is a sketch for " a formulation in six variances" , bringing together the common features of a duality formalism (inspired by those of Poincaré and Serre) which I drew up between 1956 and 1963, a 'formula' which lays claim to being 'universal' for every situation of cohomological duality encountered up to the present day. This formulation appears to have fallen into oblivion after my departure from the mathematical world, to the extent that, to my knowledge and apart from myself, no one has bothered even to draw up a list of the fundamental operations, those fundamental canonical isomorphisms which they engender, and the resemblances between them.

This sketch for a coherent formulation would turn out to be for me the first step towards that "grand delineation of the dream of the motives " which, for more than 15 years, " awaits the bold mathematician who would dare to tackle it". It appears to be the case that this mathematician has to be me. Indeed it is high time that this notion, born in my private reflections over twenty years ago, which was never intended to be the property of a single person but was destined for all, should finally emerge from the obscurity of night, to be born once again in the full light of day.

It is the case of course that there's only one person, apart from myself, who has developed an intimate knowledge of this 'yoga of motives' , who in fact learned about it directly from me in the days and years preceding my departure. Among all the mathematical discoveries which I've been privileged to make, the concept of the motive still impresses me as the most fascinating, the most charged with mystery - indeed at the very heart of the profound identity of geometry and arithmetic . And the yoga of motives which brought me to this now much neglected reality is perhaps the most powerful research tool invented by me during the first period of my life as a mathematician.

Yet it is also true that this reality, and this "yoga" that closely surrounds it, were never kept as personal secrets. Absorbed as I was in the urgent task of communicating the fundamentals (which since then everyone else is happily content to use in their daily work) , I could not find the extra months needed to edit an enormous sketch I'd drawn up about the yoga of motives in its totality, thereby putting it at the disposal of everyone. But I did not fail, in the years before my departure, to speak about them at conferences or to anyone who cared to listen to me, beginning with my students who (with the exception of one of them), have totally forgotten everything I taught them, just like everyone else.

If I speak of them now, it is not with the intention of augmenting the list of 'inventions' that bear my name, but rather to draw attention on a mathematical reality which is virtually self-evident once one interests oneself in the cohomology of algebraic varieties, particularly of their arithmetical properties, and of their relationship to all other cohomological theories current at the present time. This reality is as concrete as, in the past, the notions of infinitely small entities were, which were understood and used long before a rigorous mathematical formalism officially established their legitimacy. And, to understand the reality of motives we are not in any short supply of a flexible language for describing them, nor do we, like our predecessors, lack experience in building mathematical theories.

Although all that I have shouted from the roof tops has, up to the present time, fallen on deaf ears, and although my disdainful elective mutism has received, as an echo, the silence and laziness of all those who claim to be interested in cohomology (and who all the same have hands and eyes just like mine) , I cannot hold alone responsible that person who has chosen to guard for himself as a kind of personal treasure all that I'd confided in him that was intended to benefit everybody. One is however forced to the conclusion that our age, whose frenzied productivity in the domain of science is at a level with that of weapons and the consumption of material goods, is at the same time a long way from that sort of "bold dynamism" of our 17th century predecessors, who did not wait to receive support from the four corners of the earth before develop an infinitesimal calculus, or worry about whether what they were doing was rigorous or pure conjecture, or wait for some eminence among them to give them the green light.They were not afraid to grasp and work with that which everybody could see first-hand with their own eyes.

9. By virtue of its inner structure and the nature of its theme, the "Funeral Rites" (which now forms more than half of the text of Récoltes et Semailles is largely independent from the long reflection that precedes it. This independence is however only apparent. For myself this meditation on a "burial" that continues to emerge bit-by-bit from the fog of the unspoken, is inseparable from what comes before, from which it grew, and

which gives it all its meaning. Begun as a quick glance, in passing, on the trials of a life's work which I'm grown somewhat out of touch with, it became, without any intention on my part, a meditation on one of the important relationships in my life, leading me in turn to another disquisition on the fate of this work in the hands of "those who were my students". To make a separation between this work and that from which it has spontaneously emerged seems to me to be a kind of simplistic reduction of this reflection to a kind of Bildungsroman (which might even be misinterpreted, in that wonderful world of mathematics, as a kind of 'settling of accounts')

It becomes such if one thinks of it in that way: a similar interpretation as a "novel of manners" could be made of the whole of Récoltes et Semailles. It is certainly the case that the customs prevailing in any given age, in a certain milieu, which shape the lives of the people who participate in it, are important and ought to be identified. However it should be clear to any careful reader of Récoltes et Semailles that I've not set out to describe manners or customs, that is to say to depict a certain kind of 'scene', one that changes with time and place, which serves as a backdrop for our acts. To some extent this backdrop defines and restricts the means at the disposal of whatever there is in us that we would wish to express. Although the setting and its opportunities (as well as its "rules of the game") are infinitely varied, the forces deep within us that (at the collective level) shape the settings and which (at the personal level), are expressed within these settings, seem to be essentially unvarying from one culture to another or from one age to the next. If there is one thing in my life, apart from mathematics and a love of women, that has evoked for me a sense of supreme mystery (quite late it's true) , it is indeed the hidden nature of these forces which determine our actions, for better or worse, for destruction or creation.

10. The reflection which eventually took the name of "The Funeral Rites" had been initiated as a token of respect : respect for the things I'd discovered, that I'd watched in the process of formation from the void, of which I was the first to savor and to appreciate the strength, to which I'd given names to express the way in which I came to know them, and, as already stated, to show my respect for them. These are the things to which I gave the best of myself, nourished by my interior force. They came out of the earth and blossomed forth like the many branches of the same tree trunk, bristling with many vigorous roots. They continue to live, not only as inventions that one might chose to make or not to make - but all closely connected and united in one vital whole that is formed from each and gives each its place and sense, its origin and its end. They were abandoned long ago, without anxiety or regret, because I knew that what I was leaving

was healthy and strong and had no more need of me to flourish, to be fruitful and to multiply, following its proper nature. It is not a mere sack of gold coins that I leave, that anyone can steal, nor a bundle of tools that can rust and decay.

However, over the course of many years, during which I felt myself distanced from the world I'd left, there came to me in my retreat, faint suggestive indications of a spirit of insidious contempt and discrete derision attached to things that I knew to be strong and beautiful, having their proper place, functioning, and irreplaceable. I had the feeling that I'd left a family of orphans to cope alone against a hostile world, a world sick with the disease of contempt, attacking all things unable to fend for themselves. It was in this state of mind that I began this disquisition, as a token of respect towards these things and towards myself - responding to the pull of a deep bond between these things and myself. He who affects to denigrate any one of these things which have been nourished by my love, is in a sense denigrating me as well, and everything that had issued from me.

And the same thing may be said of someone who, knowing at first hand of the tie that unites me to certain things which he learned from none other than me, would pretend that this tie was of no significance, or to claim to be unaware of its existence, or to make claim (either overtly or through omission) on his account or that of someone else, of a fictive "paternity". These clearly represent, to my mind, acts of contempt for the products of my labor, as well as the obscure and subtle work involved in bringing these things to birth, and to me, the worker himself, and indeed, (in a more hidden yet essential way), that individual himself.

If my "return to mathematics" will have no other effect than that of making me once more aware of that bond, and in bringing out this token of my respect to the awareness of others - those who affect contempt and those who pretend to indifference - it will not have been in vain.

It is true enough that I'd really lost contact with the works (written and unwritten, or at least unpublished) , that I'd left behind. When I started this reflection I saw all the branches distinctly, without remembering how they were all connected up to the same tree. Strangely enough, it took the gradual revelation of a spectacle of pillaging of what I'd left, for me to discover the vital unity of that which has been ruthless plundered and dispersed. One took the bag of gold coins, another a tool or two from the toolbox, to claim credit for himself or to exploit in some other way. Yet the unity which gave life and the true force behind what I'd left has totally escaped all of them.

Still, I know of a single person who was deeply touched by this unity and this force, and, in his inner depths still feels it, whom it pleases to disperse this force and who has it in him to destroy this unity which he discovered in another (by the intermediary of his works) . It is in this living unity that one finds the beauty and the creative virtue of this opus. Notwithstanding the pillage, I find them intact, just as they were when I left, though I have matured and see them today with new eyes.

If I find that these things have been despoiled and mutilated, robbed of their initial force, I know that it has been done by those unaware of their own interior force, who think they can get away with despoiling at their pleasure. Yet all they are doing is cutting themselves off from the creative virtue which is at their disposal as it is at the disposal of everyone, yet never at the mercy or power of another.

Thus this reflection, and, through it the unexpected "return", has led me to reestablishing contact with a forgotten beauty. It is the sensation of this beauty which gives meaning to this act of respect, but poorly expressed in the note "My orphans"(*), which I invoke now, in full recognition of its significance.

(*) This note (#46) is chronologically the first to figure in "The Funeral Rites" .

1. The Magic of Things

When I was a child I loved going to school. The same instructor taught us reading, writing and arithmetic, singing (he played upon a little violin to accompany us), the archaeology of prehistoric man and the discovery of fire. I don't recall anyone ever being bored at school. There was the magic of numbers and the magic of words, signs and sounds. And the magic of rhyme, in songs or little poems. In rhyming there appeared to be a mystery that went beyond the words. I believed this until the day on which it was explained to me that this was just a 'trick': all one had to do in making a rhyme was to end two consecutive statements with the same syllable. As it by miracle, this turned ordinary speech into verse. What a revelation! In conversations at home I amused myself for weeks and months in spontaneously making verses. For awhile everything I said was in rhyme. Happily that's past. Yet even today, every now and then, I find myself making poems - but without bothering to search for rhymes when they do not arise spontaneously.

On another occasion a buddy who was a bit older than me, who was already going to the primary school , instructed me in negative numbers. This was another amusing game, yet one which lost its interest more quickly. And then there were crossword puzzles. I passed many a day in making them up, making them more and more complicated. This particular game combined the magic of forms with those of signs and words. Yet this new passion also passed away without a trace.

I was a good student in primary school, in Germany for the first year and then in France, although I wasn't what would be considered 'brilliant'. I became thoroughly absorbed in whatever interested me , to the detriment of all else, without concerning myself with winning the appreciation of the teacher. For my first year of schooling in France, 1940., I was interned with my mother in a concentration camp, at Rieucros, near Mende. It was wartime and we were foreigners - "undesirables" as they put it. But the camp administration looked the other way when it came to the children in the camp, undesirable or not. We came and left more or less as we wished. I was the oldest and the only one enrolled in school. It was 4 or 5 kilometers away, and I went in rain, wind and snow, in shoes if I was lucky to find them, that filled up with water.

I can still recall the first "mathematics essay", and that the teacher gave it a bad mark. It was to be a proof of "three cases in which triangles were congruent ". My proof wasn't the official one in the textbook he followed religiously. All the same, I already knew that my proof was neither more nor less convincing than the one in the book, and that it was in accord with the traditional spirit of "gliding this figure over that one". It was self-evident that this man was unable or unwilling to think for himself in judging the worth of a train of reasoning. He needed to lean on some authority, that of a book which he held in his hand. It must have made quite an impression on me that I can now recall it so clearly. Since that time, up to this very day, I've come to see that personalities like his are not the exception but the rule. I have lots to say about that subject in Récoltes et Semailles. Yet even today I continue to be stunned whenever I confront this phenomenon, as if it were for the first time.

During the final years of the war, during which my mother remained interned, I was placed in an orphanage run by the "Secours Suisse", at Chambon sur Lignon. Most of us were Jews, and when we were warned (by the local police) that the Gestapo was doing a round-up, we all went into the woods to hide for one or two nights, in little groups of two or three without concerning ourselves overmuch if it was good for our health. This region of the Cévennes abounded with Jews in hiding. That so many survived is due to the solidarity of the local population.

What struck me above all at the "Collège Cévenol" (where I was enrolled) was the extent to which my fellows had no interest in anything they were learning. As for myself I devoured all of my textbooks right from the beginning of each school year, convinced that this year, at last, we were really going to learn really interesting. Then for the rest of the year I had to figure out ways to employ my time as the program unfolded itself with tedious slowness over the course of the semester. However I should say that there were some really great teachers. Monsieur Friedal our instructor for Biology, was a man of high personal and intellectual qualities. However he was totally incapable of administering discipline, so that his class was in an interminable turmoil. So loud was the ruckus that it was impossible hear his voice rising above the din. No doubt that explains why I didn't become a biologist!

Much of my time, even during my lessons, (shh!..) was spent working on math problems. It wasn't long before the ones I found in the textbook were inadequate for me. This may have been because they all tended to resemble each other; but mostly because I had the impression that they were plucked out of the blue, without any idea of the context in which they'd emerged. They were 'book problems', not 'my problems'. However, there were questions that arose naturally. For example, when the lengths a,b,c of the three sides of a triangle are known, then the triangle itself is determined (up to its position in space), therefore there ought to be some explicit formula for expressing the area of that triangle as a function of a, b and c. The same had to be true for a tetrahedron when the 6 sides are known: what is its volume? That caused me no little difficulty, but in the end I did derive the formula after a lot of hard work. At any rate, once a problem "grabbed me", I stopped paying attention to the amount of time I had to spend on it, nor of all the other things that were being sacrificed for its sake (This remains true to this day).

What I found most unsatisfactory in my mathematics textbooks was the absence of any serious attempt to tackle the meaning of the idea of the arc-length of a curve, or the area of a surface or the volume of a solid. I resolved therefore to make up for this defect once I found time to do so. In fact I devoted most of my energy to this when I became a student at the University of Montpellier, between 1945 and 1948. The courses offered by the faculty didn't please me in the least. Although I was never told as much, I'd the impression that the professors had gotten into the habit of dictating from their texts, just like they used to do in the lycée at Mende. Consequently I stopped showing up at the mathematics department, and only did so to keep in touch with the official 'program'. For this purpose the textbooks were sufficient, but they had little to do with the questions I was posing, To speak truthfully, what they lacked was insight , even as the textbooks in the lycée were lacking in insight . Once delivered of their formulae for calculating lengths, areas, volumes in terms of simple, double or triple integrals (higher dimensions carefully avoided), they didn't care to probe further into the intrinsic meaning of these things. And this was as true of my professors as it was of the books from which they taught.

On the basis of my very limited experience I'd the impression that I was the only person in the entire world who was curious to know the answers to such mathematical questions. That was, at least, my private and unspoken opinion during all those years passed in almost total intellectual isolation, which, I should say, did not oppress me overmuch.(*) I don't think I ever gave any deep thought to trying to find out whether or not I was the only person on earth who considered such things important. My energies were sufficiently absorbed in keeping the promise I'd made with myself: to develop a theory that could satisfy me.

(*)Between 1945 and 1948 my mother and I lived in a small hamlet about a dozen kilometers from Montpellier, named Mairargues (near Vendargues), surrounded by vineyards. (My father disappeared in Auschwitz in 1942). We lived marginally on the tiny government stipend guaranteed to college students in France. Each year I participated in the grape harvests ("vendanges". Translators Note: I worked in these briefly, in the summer of 1970, in the region around Dijon.). After the harvests there was the gathering up of the loose remains of the grapes in the fields (grapillage), from which we made a more or less acceptable wine (apparently illegally) . There was in addition, our garden, which, without having to do much work in it, furnished us with figs, spinach and even (in the late Fall) tomatoes, which had been planted by a well-disposed neighbor right in the middle of a splendid field of poppies. It was 'the good life' . although a little on the short side when it came to getting a new pair of glasses, or having to wear out one's shoes down to the soles. Luckily my mother, chronically invalided from her long term in the internment camps, had the right to free medical care. There was no way we could have paid for doctors.

I never once doubted that I would eventually succeed in getting to the bottom of things, provided only that I took the effort to thoroughly review the things that came to me about them, and which I took pains to write down in black and white. We have, for example, an undeniable intuition of volume . It had to be the reflection of some deeper reality , which for the moment remained elusive, but was ultimately apprehensible. It was this reality, plain and simple, that had to be grasped - a bit, perhaps, the way that the "magic of rhyme" had been grasped one day in a moment of understanding.

In applying myself to this problem at the age of 17 and fresh out of the lycée, I believed that I could succeed in my objective in a matter of weeks. As it was, it preoccupied me fully three years. It even led me to flunk an examination, during my second year in college - in spherical trigonometry! (for an optional course on 'advanced astronomy') because of a stupid mistake in arithmetic. (I should confess here that I've always been weak in arithmetic, ever since leaving the lycée .)

Because of this I was forced to remain for a third year at Montpellier to obtain my license(*)

(*)Translator's Note: the basic undergraduate degree in the French university system, not quite the same as our B.A.)

, rather than heading immediately up to Paris - the only place, I was told, where one found people who really knew what was important in modern mathematics. The person who said this to me, Monsieur Soula, also assured me that all outstanding issues in mathematics had been stated and resolved, twenty or thirty years before, by a certain " Lebesgue" ! (Translator's Italics). In fact he'd developed a theory of integration and measure (decidedly a coincidence!), beyond which nothing more needed to be said.

Soula, it should be said, was my teacher for differential calculus, a good-hearted man and well disposed towards me. But he did not succeed at all in persuading me to his point of view. I must already have possessed the conviction that Mathematics has no limit in grandeur or depth. Does the sea have a "final end" ? The fact remains that at no point did it occur to me to dig out the book by Lebesgue that M. Soula had recommended to me, which furthermore he himself had never looked at! To my point of view, I could see little connection between what one might find in a book and the work I was doing to convince my own curiosity on issues that perplexed and intrigued me.

2.The importance of Solitude

A few years after I finally established contact with the world of mathematics at Paris, I learned, among other things, that the work I'd in my little niche with the means at my disposal had (essentially) been long known to the whole world under the name of "Lebesgue's theory of measure and integration". In the eyes of my mentors, to whom I'd described this work, and even shown them the manuscript, I'd simply "wasted my time", merely doing over again something that was "already known". But I don't recall feeling any sense of disappointment. At that time the very notion of "taking credit" for my own work, either to receive compliments or even the mere interest of anyone else,) was furthest from my thoughts. My energies at that time were completely taken up with adjusting to a totally unfamiliar environment, above all with learning what one had to know to be treated like a mathematician.(*)

(*)I talk briefly about this transitional period, which was rather rough, in the first part of Récoltes et Semailles (R&S I), in the section entitled "Welcoming the Stranger" (#9)

However, re-thinking those three years (1945-48), I realize that they weren't wasted in the least. Without recognizing it, I'd thereby familiarized myself with the conditions of solitude that are essential for the profession of mathematician , something that no-one can teach you. Without having to be told, without having to meet others who shared my thirst for understanding, I already knew "in my guts", that I was indeed a mathematician: because I knew that I was one who "makes mathematics", in the way someone "makes love". Quite simply, mathematics had become a mistress, ever receptive to gratifying my desire. These years of isolation laid the foundation for a faith that has never been shaken - neither by the discovery (arriving in Paris at the age of 20), of the full extent of my ignorance and the immensity of what I would be obliged to learn; nor (20 years later) by the turbulent events surrounding my final departure from the world of mathematics; nor, in recent years, by the thoroughly weird episodes of a metaphorical "Burial" of my person and my work, so perfectly orchestrated by those who were formerly my closest friends ....

To state it in slightly different terms: in those critical years I learned how to be alone (*)

(*) This formulation doesn't really capture my meaning. I didn't, in any literal sense learn to be alone, for the simple reason that this knowledge had never been unlearned during my childhood. It is a basic capacity in all of us from the day of our birth. However these 3 years of work in isolation, when I was thrown onto my own resources, following guidelines which I myself had spontaneously invented, instilled in me a strong degree of confidence, unassuming yet enduring, in my ability to do mathematics, which owes nothing to any consensus or to the fashions which pass as law. I come back to this subject again in the note: "Roots and Solitude" (R&S IV, #171.3, in particular page 1080).

By this I mean to say: to reach out in my own way to the things I wished to learn, rather than relying on the notions of the consensus, overt or tacit, coming from a more or less extended clan of which I found myself a member, or which for any other reason laid claim to be taken as an authority. This silent consensus had informed me, both at the lyé and at the university, that one shouldn't bother worrying about what was really meant when using a term like "volume", which was "obviously self-evident", "generally known", "unproblematic", etc. I'd gone over their heads, almost as a matter of course, even as Lesbesgue himself had, several decades before, gone over their heads. It is in this gesture of "going beyond", to be something in oneself rather than the pawn of a consensus, the refusal to stay within a rigid circle that others have drawn around one - it is in this solitary act that one finds true creativity. All others things follow as a matter of course.

Since then I've had the chance, in the world of mathematics that bid me welcome, to meet quite a number of people, both among my "elders" and among young people in my general age group, who were much more brilliant, much more "gifted" than I was. I admired the facility with which they picked up, as if at play, new ideas, juggling them as if familiar with them from the cradle - while for myself I felt clumsy. even oafish, wandering painfully up a arduous track, like a dumb ox faced with an amorphous mountain of things that I had to learn (so I was assured), things I felt incapable of understanding the essentials or following through to the end. Indeed, there was little about me that identified the kind of bright student who wins at prestigious competitions or assimilates, almost by sleight of hand, the most forbidding subjects.

In fact, most of these comrades who I gauged to be more brilliant than I have gone on to become distinguished mathematicians. Still, from the perspective of 30 or 35 years, I can state that their imprint upon the mathematics of our time has not been very profound. They've all done things, often beautiful things, in a context that was already set out before them, which they had no inclination to disturb. Without being aware of it, they've remained prisoners of those invisible and despotic circles which delimit the universe of a certain milieu in a given era. To have broken these bounds they would have had to rediscover in themselves that capability which was their birth-right, as it was mine: the capacity to be alone.

The infant has no trouble whatsoever being alone. It is solitary by nature, even when it's enjoying the company surrounding him or seeks his mother's tit when it is in need of it. And he is well aware, without having to be told, that the tit is for him, and knows how to use it. Yet all too often we have lost touch with the child within us. And it's often the case that we pass by the most important things without bothering to look at them...

If, in Récoltes et Semailles I'm addressing anyone besides myself, it isn't what's called a "public". Rather I'm addressing that someone who is prepared to read me as a person , and as a solitary person. It's to that being inside of you who knows how to be alone, it is to this infant that I wish to speak, and no-one else. I'm well aware that this infant has been considerably estranged. It's been through some hard times, and more than once over a long period. It's been dropped off Lord knows where, and it can be very difficult to reach. One swears that it died ages ago, or that it never existed - and yet I am certain it's always there, and very much alive.

And, as well, I know how to recognize the signs that tell me I'm being understood. It's when, beyond all differences of culture and fate, what I have to say about my person finds an echo and an resonance in you, in that moment when you see , your own life , your own experience, in a light which, up to that moment, you'd not thought of paying attention to. It's not a matter of some sort of "re-identifying " something or someone that was lost to you. It means that you have rediscovered your own life, that which is closest to you, by virtue of the rediscovery that I've made of mine in the course of my writing these pages of R8#233coltes et Semailles, and even in those pages that I am in the process of setting down at this very moment.

3. The Interior Adventure - or Myth and Witnessing

Above all else, Récoltes et Semailles is a reflection on myself and only my life. At the same time, it is also a testimonial ,and this in two ways. The testimonial on my past takes up the major portion of this reflection. Yet at the same time it is a testimonial to my immediate present - that is to say, up to the moment at which I'm writing it, in which the pages of Récoltes et Semailles are taking shape by the hour, night and day. These pages are the faithful witnesses of this long meditation on my life, as it is unfolding in real time, (and as it is unfolding even at this actual moment....)

These pages make no claim to literary excellence. They should be seen as a form of documentation on myself. I have refrained from touching it up in any way, (certainly not for stylistic reasons), save in a very restricted sense(*)

(*) Thus, the rectification of mistakes (factual or interpretive) are not revised in the draft itself but appear as footnotes at the bottom of the page, or on those occasions when I return to the discussion of an earlier subject matter.

If there is any affection on my part, it is the affectation of speaking the truth. And that's already quite a lot

Furthermore one shouldn't look upon this document as some kind of "autobiography". You won't learn anything about my date of birth (which can only be of interest to someone engaged in casting my horoscope), nor the names of my father nor mother, or what they did in their lives, nor the name of my wife, or of other women who've been important in my life, or that of the children born from these loves, or what any of these people have done with their lives. It's not that these things haven't had their importance in my life, or have lost any of their importance. It is only that from the moment I began to work on this reflection I've felt under no compulsion to talk about these things directly, simply touching on them from time to time when they became relevant, nor have I felt impelled to cite names or vital statistics. It has never been my impression that doing so would add something meaningful to whatever I was engaged in examining at one time or another. (Thus, in the small selection of pages preceding this one I've included more of such details than in the 1000 pages that follow it.

And, if you want to know, what is the "proposal" that I've laid out in over a thousand pages, my reply is: to tell the story, and by doing so to make the discovery of the interior adventurewhich has been and which continues to be the story of my life. This documentation-testament of my adventure is being conducted simultaneously on the two levels that I've speak about. There is first of all an exploration of the past adventure, its roots and origins in my childhood. And, secondly, there is the continuation and the rejuvenation of that "same" adventure, in line with the days and even the instants of the composition of Récoltes et Semailles, as a spontaneous response to a violent provocation into my life coming from the external world.(**)

External events enrich this reflection only to the extent that they arouse a return to the interior adventure, or contribute to its clarification. Such a provocation has arisen from the long standing burial and plundering of my mathematical opus. It has aroused in my very powerful reactions of an unabashedly egocentric character, while at the same time revealed to me the profound ties which, unbeknownst to me continue to bind me to my opus

The fact that I happen to be one of the strong figures in modern mathematics does not, it is true, supply any reason why others should find my interior adventure interesting; nor does the fact that I'm on the outs with my colleagues after having totally changed my social environment and life style. Besides, there are any number of these colleagues, and even supposed friends, who don't hesitate, in public, to ridicule my so-called 'spiritual states'. What counts to them are 'results' and nothing else. The "soul", (which is to say that entity within us which experiences the "production" of these "results" , or its direct effects , (such as the life of the "producer" , as well as that of his associates)) is systematically despised, often with overtly promulgated derision. Such attitudes are often labeled "humility" ! To me this is merely a symptom of denial, of a strange sort of alienation , present in the very air we all breath. It is a certainty that I don't write for the kind of person afflicted with this sort of disdain, who presumes to denigrate that which is the very best of what I have to offer him. A disdain, moreover, for what in fact determines his own life, as it has determined mine: those movements, superficial or profound, gross or subtle that animate the psyche, that very "soul" which lives experience and reacts upon it, which congeals or evaporates, which withdraws into itself or opens up ...

The recital of an interior adventure can only be made by he who has lived it, and by none other. But, even if this recital has only been intended for one's own benefit, it is rare that it doesn't fall into the category of myth whose hero is the narrator. Such myths are born, not from the creative imagination of a culture or a people. but merely from the vanity of somehow who dare not accept a humbling reality, who has substituted for this reality some self-conceived fabrication. However, a true account,

(if it is so) of an interior adventure as it has been truly lived is a precious thing. Not because of the prestige (rightly or wrong) that surrounds the narrator, but solely from the fact that something with that degree of truthfulness really exists. Such a testament is priceless, whether it comes from a person deemed illustrious or notorious, or from some insignificant wage earner responsible for his family with little hope for the future, or even from a common criminal.

If this recitation of the facts has any value for others, it is to make them come face-to-face with their own selves, by means of an unvarnished testament of someone else's experience. Or, to state the case differently, to efface in himself, (even in the short time that it takes to read it), the contempt he holds for his own adventure, and for that "soul" which is both the passenger and the pilot .

4. The Novel of Manners

In speaking of my mathematical past, and in the course of doing so uncovering (as if it were a matter of rescuing my own body) the mysterious turns taken by the colossal Burial of my life's work I have been led, without having intended it, to draw up a portrait of a certain milieu in a certain time in history - a time marked by the disintegration of certain timeless values which give meaning to all human endeavor. This is the aspect of the 'novel of manners', developing around a historical event which in no doubt unique in the " Annals of "ScienceÓ. What has already been stated must make it clear that one shouldn't expect to find in Récoltes et Semailles, the "police report" or "dossier" of some celebrated "affair", written solely for the purpose of bringing one up to date. Any friend looking for such a report will go through it with his eyes closed, having seen nothing of any of the flesh and blood substance of Récoltes et Semailles.

As I explain, in much greater detail, in The Letter, the "police investigation" (or the "novel of manners") is to be found principally in Parts II and IV: "The Burial (1)-or the Robe of the Emperor of ChinaÓ, and: "The Burial (3) -or the Four Operations". In the course of writing these pages I have stubbornly brought to light a multitude of "juicy" findings, (to say the least), which I've attempted, for better or for worse, to "spruce up". Bit by bit I've found a coherent picture slowly emerging from the mists, one whose colors grow in intensity, one whose contours are becoming progressively sharper. In the notes that I've made on a daily basis, the "raw facts" which surface are inextricably mixed with personal reminiscences, comments and reflections on psychology, philosophy and even mathematics. That's the way it is and I can't do anything about it!

On the basis of the work already done, which has absorbed me for over a year, anyone wishing to extract a "dossier", in the mode of an investigative "wrap-up", will have to spend many additional hours, if not days, depending on the interest or curiosity of the reader, in working it out. At one point I myself tried to extract such a dossier. This when I began the long footnote now known as "The Four Operations"(*)

(*) What was intended as a footnote exploded into all of Part IV (with the same title of "The Four Operations"), comprising 70 notes stretching over 400 pages.

Ultimately it wasn't possible. I failed totally! It's not my style, certainly not in my elderly years. In my present estimation, I've done enough, with the production of Récoltes et Semailles, for the benefit of the mathematics community to be able, without regrets, to leave for others (who may perhaps be found among my colleagues) the work of putting together the dossier it contains.

5. The Inheritors and the Builder

The time has come to say a few words about my work in mathematics, something which at one point I held (and to my surprise still is) to be of some importance. I return more than once in Récoltes et Semailles to consider that work, sometimes in a manner that ought to be clear to everyone, though at other times in highly technical terms.(*)

(*)Once in awhile one will discover, in addition to my observations about my past work, a discussion of some contemporary mathematical developments. The longest among these is in "The 5 photographs (Crystals and D-Modules)" in R&S IV, note #171 (ix)

The latter passages will no doubt, for the most part, be 'over the heads' not only of the lay public, but also of those mathematical colleagues who aren't involved in this particular branch of mathematics. You are certainly more than welcome to skip any passages which impress you as being too 'specialized'. Yet even the layman may want to browse them, and by doing so perhaps be taken by the sense of a 'mysterious beauty' (as one of my non-mathematician friends has written) moving about within them like so many "strange inaccessible islands" in the vast and churning occasions of thought.

As I've often said, most mathematicians take refuge within a specific conceptual framework, in a "Universe" which seemingly has been fixed for all time - basically the one they encountered "ready-made" at the time when they did their studies. They may be compared to the heirs of a beautiful and capacious mansion in which all the installations and interior decorating have already been done, with its living-rooms , its kitchens, its studios, its cookery and cutlery, with everything in short, one needs to make or cook whatever one wishes. How this mansion has been constructed, laboriously over generations, and how and why this or that tool has been invented (as opposed to others which were not), why the rooms are disposed in just this fashion and not another - these are the kinds of questions which the heirs don't dream of asking . It's their "Universe", it's been given once and for all! It impresses one by virtue of its greatness, (even though one rarely makes the tour of all the rooms) yet at the same time by its familiarity, and, above all, with its immutability.

When they concern themselves with it at all, it is only to maintain or perhaps embellish their inheritance: strengthen the rickety legs of a piece of furniture, fix up the appearance of a facade, replace the parts of some instrument, even, for the more enterprising, construct, in one of its workshops, a brand new piece of furniture. Putting their heart into it, they may fabricate a beautiful object, which will serve to embellish the house still further.

Much more infrequently, one of them will dream of effecting some modification of some of the tools themselves, even, according to the demand, to the extent of making a new one. Once this is done, it is not unusual for them make all sorts of apologies, like a pious genuflection to traditional family values, which they appear to have affronted by some far-fetched innovation.

The windows and blinds are all closed in most of the rooms of this mansion, no doubt from fear of being engulfed by winds blowing from no-one knows where. And, when the beautiful new furnishings, one after another with no regard for their provenance, begin to encumber and crowd out the space of their rooms even to the extent of pouring into the corridors, not one of these heirs wish to consider the possibility that their cozy, comforting universe may be cracking at the seams. Rather than facing the matter squarely, each in his own way tries to find some way of accommodating himself, one squeezing himself in between a Louis XV chest of drawers and a rattan rocking chair, another between a moldy grotesque statue and an Egyptian sarcophagus, yet another who, driven to desperation climbs, as best he can, a huge heterogeneous collapsing pile of chairs and benches!

The little picture I've just sketched is not restricted to the world of the mathematicians. It can serve to illustrate certain inveterate and timeless situations to be found in every milieu and every sphere of human activity, and (as far as I know) in every society and every period of human history. I made reference to it before , and I am the last to exempt myself: quite to the contrary, as this testament well demonstrates. However I maintain that, in the relatively restricted domain of intellectual creativity, I've not been affected (*) by this conditioning process, which could be considered a kind of 'cultural blindness' ?an incapacity to see (or move outside) the "Universe" determined by the surrounding culture.

(*) The reasons for this are no doubt to be found in the propitious intellectual climate of my infancy up to the age of 5. With respect to this subject look at the note entitled " Innocence", (R&S III,# 107) .

I consider myself to be in the distinguished line of mathematicians whose spontaneous and joyful vocation it has been to be ceaseless building new mansions. (**)

(**)This archetypal image of the "house" under construction appears and is elaborated for the first time in the note "Yin the Servant, and the New Masters" (R&S III #135)

We are the sort who, along the way, can't be prevented from fashioning, as needed, all the tools, cutlery, furnishings and instruments used in building the new mansion, right from the foundations up to the rooftops, leaving enough room for installing future kitchens and future workshops, and whatever is needed to make it habitable and comfortable. However once everything has been set in place, down to the gutters and the footstools, we aren誰 the kind of worker who will hang around, although every stone and every rafter carries the stamp of the hand that conceived it and put it in its place.

The rightful place of such a worker is not in a ready-made universe, however accommodating it may be, whether one that he's built with his own hands, or by those of his predecessors. New tasks forever call him to new scaffoldings, driven as he is by a need that he is perhaps alone to fully respond to. He belongs out in the open. He is the companion of the winds and isn't afraid of being entirely alone in his task, for months or even years or, if it should be necessary, his whole life, if no-one arrives to relieve him of his burden. He, like the rest of the world, hasn't more than two hands - yet two hands which, at every moment, know what they're doing, which do not shrink from the most arduous tasks, nor despise the most delicate, and are never resistent to learning to perform the innumerable list of things they may be called upon to do. Two hands, it isn't much, considering how the world is infinite. Yet, all the same, two hands, they are a lot ....

I'm not up on my history, but when I look for mathematicians who fall into the lineage I'm describing, I think first of all of Evariste Galois and Bernhard Riemann in the previous century, and Hilbert at the beginning of this one. Looking for a representative among my mentors who first welcomed me into the world of mathematics (*), Jean Leray's name appears before all the others, even though my contacts with him have been very infrequent. (**)

(*)I talk about these beginnings in the section entitled "The welcome stranger"(ReS I, #9)

(**)Even so I've been (following H. Cartan and J.P.Serre), one of the principal exploiters and promoters of one of the major ideas introduced by Leray, that of the bundle . It has been an indispensable tool in all of my work in geometry. It also provided me with the key for enlarging the conception of a (topological) space to that of a topos, about which I will speak further on.

Leray doesn't quite fill this notion that I have of a 'builder', in the sense of someone who 'constructs houses from the foundations up to the rooves." However, he's laid the ground for immense foundations where no one else had dreamed of looking, leaving to others the job of completing them and building above them or, once the house has been constructed, to set themselves up within its rooms (if only for a short time) .... .

I've used large brush strokes in the making of my two sketches: that of the 'homebody" mathematician who is quite happy in adding a few ornaments to an established tradition, and that of the pioneer-builder(*), who cannot be restrained from crossing the 'imperious and invisible boundaries' that delimit a Universe (**)

(*)Convenience has led me to form this hyphenated compound with a masculine resonance , "pioneer-builder' ("batisseur", and "pionnier"). These words express different phases in the impulse towards discovery whose connections are in fact too delicate to be satisfactorily expressed by them. A more satisfactory discussion will appear following this 'walking meditation', in the section "In search of the mother-or the two aspects' (#17)

(**)Furthermore, at the same time, and without intending to, he assigns to the earlier Universe (if not for himself then at least for his less mobile colleagues), a new set of boundaries, much enlarged yet also seemingly imperious and invisible than the ones he's replaced

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One might also call them using names that are perhaps less appropriate yet more suggestive, the "conservators" and the "innovators" . Both have their motivations and their roles to play in the same collective adventure that mankind has been pursuing over the course of generations, centuries and millennia. In periods when an art or a science is in full expansion, there is never any rivalry between these two opposing temperaments(***). They differ yet are mutually complementary, like dough and yeast.

Such was the situation in mathematics during the period 1948-69 which I personally witnessed, when I was myself a part of that world. A period of reaction seems to have set in after my departure in 1970, one might call it a "consensual scorn" for 'ideas' of any sort, notably for those which I had introduced.

Between these two types at the extremes (though there is no opposition in nature between them) one finds a spectrum of every kind of person. A certain "homebody" who cannot imagine that he will ever leave his familiar home territory, or even contemplate the work involved in setting up somewhere else, will all the same put his hand to the trowel for digging out a cellar or an attic, add on another story, even go so far as to throw up the walls for a new, more modest, building next to his present one.(****)

(****) Most of my mentors (to whom I devote all my attention in "A welcome debt", Introduction, 10), have this in-between temperament. I'm thinking in particular of Henri Cartan, Claude Chevalley, André Weil, Jean-Pierre Serre, Laurent Schwartz. With the exception perhaps of Weil, they all, at last, cast an "auspicious eye" without "anxiety or private disapproval" at the lonely adventures in which I was engaged.

Without having the character of a true builder, he will frequently express sympathy for one who does, or at least feel no anxiety or private disapproval towards one who has shared the same dwelling with him, even when he does strange things like setting up pillars and building blocks in some outlandish setting, with the attitude of someone who already sees a palace in front of him.

6.Visions and Viewpoints

But I must return to myself and my work.

If I have excelled in the art of the mathematician, it is due less to my facility or my persistence in working to find solutions for problems delegated to me by my predecessors, than to the natural propensity which drives me to envisage questions , ones that are clearly critical, which others don't seem to notice, and to come up with "good ideas" for dealing with them (while at the same time no-one else seems to suspect that a new idea has arrived), and "original formulations" which no-one else has imagined. Very often, ideas and formulations interact in so effective a manner, that the thought that they might be incorrect does not arise, (apart from touching them up a bit). Also as well, when its not a matter of putting the pieces together for publication, I take the time to go further, or to complete a proof which, once the formulation and its context have been clarified, is nothing more than what is expected of a true "practitioner", if not simply a matter of routine. Numberless things command our attention, and one simply cannot follow all of them to the end! Despite this it is still the case that the theorems and propositions in my written and published work that are cast into the proper form of a demonstration number in the thousands. With a tiny number of exceptions they have all joined the patrimony of things accepted as "known" by the community, and are used everywhere.

Yet, even more than in the discovery of new questions, notions and formulations, my unique talent appears to consist of the entertainment of fertile points of view which lead me to introduce and to, more or less, develop completely original themes. It is that constitutes my most essential contribution to the mathematics of my time. To speak frankly, these innumerable questions, notions and formulations of which I've just spoken, only make sense to me from the vantage of a certain 'point of view' - to be more precise, they arise spontaneously through the force of a context in which they appear self-evident: in much the same way as a powerful light (though diffuse) which invades the blackness of night, seems to give birth to the contours, vague or definite, of the shapes that now surround us. Without this light uniting all in a coherent bundle, these 10 or 100 or 1000 questions, notions or formulations look like a heterogeneous yet amorphous heap of "mental gadgets", each isolated from the other- and not like parts of a totality of which, though much of it remains invisible, still shrouded in the folds of night, we now have a clear presentiment.

The fertile viewpoint is that which reveals to us, as so many parts of the same whole that surrounds them and gives meaning to them, those burning questions that few are aware of, (perhaps in response to these questions) thoroughly natural notions yet which none had previously conceived, and formulations which seem to flow from a common source, which none had dared to pose despite their having been suggested for some time by these questions, and, and for which the ideas had yet to emerge. Far more indeed, than what are called the "key theorems:" of mathematics, it is these fertile viewpoints which are, in our particular craft(*) the most powerful tools for discovery- rather they are not tools exactly, but the very eyes of the researcher who, in a deeply passionate sense, wishes to understand the nature of mathematical reality.

(*) This is not only the case in "our art", but, so it seems to me, in all forms of discovery, at least in the domain of intellectual knowledge/

Thus, the fertile viewpoint is nothing less than the "eye" which, at one and the same time, enables us to discover and, at the same time, recognize the simple unitybehind the multiplicity of the thing discovered. And, this unity is, veritably, the very breath of life that relates and animates all this multiplicity.

Yet, as the word itself suggests, a "viewpoint" implies particularity. It shows us but a single aspect of a landscape or a panorama out of a diversity of others which are equally valuable, and equally "real". It is to the degree that the complementary views of the same reality cooperate, with the increasing population of such "eyes", that one's understanding of the true nature of things advances. The more complex and rich is that reality that we wish to understand, the more the necessity that there be many "eyes" (**)for receiving it in all its amplitude and subtlety.

(*)Every viewpoint entails the development of a language appropriate to itself for its expression. To "have several eyes", or several "viewpoints" for comprehending a certain situation, also requires (at least in mathematics), that one has at one's disposal several distinct languages with which to grasp it.

And it often happens that a light-beam composed of many viewpoints focusing on a single immense landscape, by virtue of that gift within us which can apperceive the One within the diversity of the Many, gives birth to something entirely new; to something which transcends each of the partial perspectives, in the same way that a living organism transcends its appendages and organs. This new thing may be named a VisionIt is vision which unites the various viewpoints that compose it, while revealing to us other viewpoints which up to then had been ignored, even as the fertile viewpoint permits one to both discover and apprehend as part of a single Unity, a multiplicity of new questions, notions and formulations.

Otherwise stated: Vision is, to the viewpoints from which it springs, and which it unites, like the clear, warm light of day is to the different frequencies of the solar spectrum. A vision that is both extensive and profound is like an inexhaustible wellspring, made to inspire and illuminate the work, not only of the person in whom it first sees the light of day and becomes its servant, but that of generations, fascinated perhaps (as he was also) by those distant boundaries which it opens up.

The Great Idea-or the Forest and the Trees

The so-called "productive period" of my mathematical activity, which is to say the part that can be described by virtue of its properly vetted publications, covers the period from 1950 to 1979, that is to say 20 years. And, over a period of 25 years, between 1945 (when I was 17), and 1969, (approaching my 42nd year), I devoted virtually all of my energy to research in mathematics. An exorbitant investment, I would agree. It was paid for through a long period of spiritual stagnation, by what one may call an burdensome oppression which I evoke more than once in the pages of Récoltes et Semailles. However, staying strictly within the limited field of purely intellectual activity, by virtue of the blossoming forth and maturation of a vision restricted to the world of mathematics alone, these were years of intense creativity

During this lengthy period of my life, the greater part of my energy was consecrated to what one might call "piece work": the scrupulous work of shaping, assembling, getting things to work, all that was essential for the construction of all the rooms of the houses, which some interior voice (a demon perhaps?) exhorted me to build, the voice of a master craftsman whispering to me now and then depending on the way the work was advancing. Absorbed as I was by the tasks of my craft- brick-layer, stone-mason, carpenter, plumber, metal worker, wood worker - I rarely had the time to write down in black and white, save in sketching the barest outlines, the invisible master-plan that except , (as it became abundantly clear later) to myself underlined everything, and which, over the course of days, months and years guided my hand with the certainty of a somnambulist.(*)

(*)The image of the "somnambulist" is inspired by the title of the remarkable book by Arthur Koestler , "The Sleepwalkers" (published in France by Calman Levy), subtitled, "A history of conceptions of the universe" from the origins of scientific thought up to Newton. An aspect of this history which particularly impressed Koestler was the extent to which, so often, the road leading from one point of out knowledge of the world to some other point, seemingly so close (and which appears in retrospect so logical), passes through the most bizarre detours almost to the point of appearing insane; and how, all the same, through these thousand-fold detours in which one appears to be forever lost, and with the certainty of "Sleepwalkers", those persons devoted to the search for the "keys" to the Universe fall upon, as if in spite of themselves and without always being aware of it, other "keys" which they did not anticipate, yet which prove in the long run to be the correct ones

On the basis of what I've been able to see around me at the level of mathematical discovery, these incredible detours of the roads of discovery are characteristic of certain great investigators only. This may be due to the fact that over the last two or three centuries the natural sciences, and mathematics even more so, have gradually liberated themselves from all the religious and metaphysical assumptions of their culture and time. which served as particularly severe brakes on the universal development, (for better or worse) of a scientific understanding of the universe. It is true, all the same, that some of the most basic and fundamental notions in mathematics (such as spatial translation, the group, the number zero, the techniques of calculus, the designation of coordinates for a point in space, the notion of a set, of a topology, without even going into negative and complex numbers), required millennia for their emergence and acceptance. These may be considered so many eloquent signs of that inherent "block", implanted in the human psyche, against the conceptualization of totally new ideas, even when these ideas possess an almost infantile simplicity, and which one would think would be obvious based on the available evidence, over generations, not to say millennia ....

To return to my own work, I've the impression that the "hand waving" (perhaps more numerous than those of my colleagues), has been largely over matters of detail. usually quickly rectified by my own careful attention. These might be called simple "accidents of the road" of a purely local character without any serious effects on the validity of the underlying intuitions of the specific situation. On the other hand, at the level of ideas and large-scale intuitions, I feel that my work stands the test of time, as incredible as that may seem. It is this certainty without hesitation of having grasped at every instant, if not exactly the ends to which my thought leads, (which often enough lie hidden), but at least the most fertile directions which ought to be explored that will lead directly to that which is most essential. It is this quality of "certitude" which has brought to my mind Koestler's image of the "sleepwalker"

It must be said that all of this piecework to which I've devoted such loving attention, was never in the least disagreeable. Furthermore, the modes of mathematical expression promoted and practiced by my mentors gave pre-eminence (to say the least!) to the purely technical aspect of the work, looking askance at any "digressions" that would appear to distract one from his narrow "motivations", that is to say, those which might have risked bringing out of the fogs some inspiring image or vision but which, because it could not be embodied right away into tangible forms of wood, stone or cement, where treated more appropriate to the stuff of dreams rather than the work of the conscientious or dedicated artisan.

In terms of its quantity, my work during these productive years found its concrete expression in more than 12,000 published pages in the form of articles, monographs or seminars(*)

(*)Starting with the 60's a portion of these publications were written in collaboration with colleagues (primarily J. Dieudonné) and students.

, and by hundreds, if not thousands of original concepts which have become part of the common patrimony of mathematics, even to the very names which I gave them when they were propounded.(**)

(**)The most significant of these ideas have been outlined in the Thematic Outline (Esquisse Thématique), and in the Historical Commentary that accompanies it, included in Volume 4 of the Mathematical Reflections. Some of their labels had been suggested to me by students or friends, such as the term "smooth morphism"(morphisme lisse) (J. Dieudonné) or the combine "site, stack, sheaf,connection" (?"site,champ,gerbe,lien") developed in the thesis of Jean Giraud.

In the history of mathematics I believe myself to be the person who has introduced the greatest number of new ideas into our science , and at the same time, the one who has therefore been led to invent the greatest number of terms to express these ideas accurately, and in as suggestive a manner as possible.

These purely "quantitative" indicators give no more, admittedly, than a crude overview of my work, to the total neglect of those things which gave it life, soul and vigor. As I've written above, the best thing I've brought to mathematics has been in terms of original viewpoints which I've first intuited,then patiently unearthed and developed bit by bit. Like the notions I've mentioned, these original viewpoints, which introduced into a great multiplicity of distinct situations, are themselves almost without limit.

However, some viewpoints are more extensive than others, which along have the capacity to encapsulate a multitude of other partial viewpoints, in a multitude of different particular instances. Such viewpoints may be characterized as "Great Ideas". By virtue of their fecundity, an idea of this kind give birth to a teeming swarm of progeny, of ideas inheriting its fertility, which , for the most part,(if not all of them) do not have as extensive a scope as the mother-concept.

When it comes to presenting a "Great Ideas", to "speak it", one is faced with, almost always, a problem as delicate as its very conception and slow gestation in the person who has conceived it - or, to be more precise, that the sum total laborious work of gestation and formation isthe "expression" of the idea: that work which consists of patiently bringing it to light, day after day, from the mists that surround its birth, to attain, little by little, some tangible form, in a picture that is progressively enriched, confirmed and refined over the course of weeks, months and years. Merely to name the idea in terms of some striking formulation, or by fairly technical key words, may end up being a matter of a few lines, or may extend to several pages. Yet it is very rare to find anyone who, without knowing it in advance, is able to "hear" this "name", or recognize its face. Then, when the idea has attained to its full maturity, one may be able to express it in a hundred or so pages to the full satisfaction of the worker in whom it had its birth . Yet it may also be the case that even a thousand pages, extensively reworked and thought over, will not suffice to capture it.(*)

(*)When I left the world of mathematics in 1970, the totality of my publications (many of which were collaborations)on the central theme of schemas came to something like ten thousand pages. This, however, constitutes only a modest portion of a gigantic programme that I envisaged about schemas. This programme was abandoned sine die with my departure, and that despite the fact that , apart from minor and inconsequential matters, everything that had already been developed and published was available to everyone, and had entered into the common heritage of notions and results normally deemed to be "well known."

That piece of my programme on the theme of schemas, their prolongations and their ramifications, that I'd completed at the time of my departure, represents all by itself the greatest work on the foundations of mathematics ever done in the whole history of mathematics(Italics added by the translator so that there should be no misunderstanding of who is speaking), and undoubtedly one of the greatest achievements in the whole history of Science.

And, in one case as in the other, among those who, in order to make it their own, have become acquainted with the work involved in bringing the idea to its full presentation, like a great forest that has miraculously sprung up in a desert- I would dare to bet that there are many among them who will, seeing all these healthy and vigorous trees, be inspired to avail themselves of them (whether for climbing, to fabricate planks and pillars, or to feed the fires in their hearths....) Yet there are few indeed who ever get to see the forest...

The Vision-or 12 Themes for a Harmonization

Perhaps one might say that a "Great Idea" is simply the kind of viewpoint which not only turns out to be original and productive, but one which introduces into a science an extraordinary and new theme. Every science, once it is treated not as an instrument for gaining dominion and power, but as part of the adventure of knowledge of our species through the ages, may be nothing but that harmony, more or less rich, more or less grand depending on the times, which unfolds over generations and centuries through the delicate counterpoint of each of its themes as they appear one by one, as if summoned forth from the void to join up and intermingle with each other.

Among the numerous original viewpoints which I've uncovered in Mathematics I find twelve which, upon reflection, I would call "Great Ideas".(*)

(*) For the sake of the mathematical reader, here is the list of these 12 master ideas, or "master-themes" of my work, in chronological order:

1. Topological Tensor Products and Nuclear Spaces
2. "Continuous" and "Discrete" dualities (Derived Categories, the "6 operations")
3. The Riemann-Roch-Grothendieck Yoga (K-Theory and its relationship to Intersection Theory)
4. Schemes
5. Topos Theory
6. Etale Cohomology and l-adic Cohomology
7. Motives, Motivic Galois Groups (*-Grothendieck categories)
8. Crystals, Crystalline Cohomology, yoga of the DeRham coefficients, the Hodge coefficients

9. "Topological Algebra":(infinity)-stacks;derivations;cohomological formalism of topoi, insipiring a new conception of homotopy.
10. Mediated topology
11. The yoga of un-Abelian Algebraic Geometry. Galois-Teichmüller Theory
12. Schematic or Arithmetic Viewpoints for regular polyhedra and in general all regular configurations. Apart from the themes in item 12, a goodly portion of which first appeared in my thesis of 1953 and was further developed in the period in which I worked in functional analysis between 1950 and 1955, the other eleven themes were discovered and developed during my geometric period, starting in 1955

To appreciate my work as a mathematician, to "sense it", is to appreciate and to sense, as best one can a certain number of its ideas, together with the grand themes they introduce which form the framework and the soul of the work.

In the nature of things, some of these ideas are "grander than others"!, others "smaller". In other words, among these new and original themes, some have a larger scope, while others delve more deeply into the mysteries of mathematical verities. (**).

(**)To give some examples, the idea of greatest scope appears to me to be that of the topos, because it suggests the possibility of a synthesis of algebraic geometry, topology and arithmetic. The most important by virtue of the reach of those developments which have followed from it is, at the present moment, the schema. (With respect to this subject see the footnotes from to the previous section(#7)) It is this theme which supplies the framework, par excellence, of 8 of the others in the above list. (that is to say, all the others except 1,5 and 10), which at the same time furnishing the central notion fundamental to a total reformation, from top to bottom, of algebraic geometry and of the language of that subject.

At the other extreme, the first and last of these 12 themes are of much less significance. However, vis-a-vis the last one, having introduced a new way of looking at the very ancient topic of the regular polyhedra and regular configurations in general, I am not sure that a mathematician who gives his whole life to studying them will have wasted his time. As for the first of these themes, topological tensor products, it has played the role of a handy tool, rather than as the springboard for future developments deriving from it. Even so I've heard , particularly in recent years, sporadic echoes of research resolving (20 or 30 years later!) some of the issues that were left open by my discoveries.

Among the 12 themes, the deepestis that of the motifs, which are closely tied to those of an-Abelian Algebraic Geometry, and that of Galois-Teichmüller Yoga .

In terms of the effectiveness of the tools I've created, laboriously polished and brought to perfection, now heavily used in certain "specialized research areas" in the last 2 decades, I would single out schemas and étale and l-adic cohomologies. For the well-informed mathematician I would claim that, up to the present moment, it can scarcely be doubted that these schematic tools. such as l-adic cohomology, etc., figure among the greatest achievements of this century, and will continue to nourish and revitalize our science in all following generations.

Among these grand ideas one finds 3 (and hardly the least among them) which, having appeared only after my departure from the world of mathematics, are still in a fairly embryonic state: they don't even exist "officially" , since they haven't appeared in any publication, (which one might consider the equivalent of a birth certificate)(*) .

(*)The only "semi-official" text in which these three themes are sketched, more or less, is the Outline for a Programme, edited in January 1984 on request from a unit of CNRS. This text (which is also discussed in section 3 of the Introduction, "Compass and Luggage"), should be, in principle, included in volume 4 of Mathematical Reflections.

The twelve principal themes of my opus aren't isolated from each other. To my eyes they form a unity, both in spirit and in their implications, in that one finds in them a single persistent tone, present in both "officially published" and "unpublished" writings.

Indeed, even in the act of writing these lines I seem to recapture that same tone- like a call! - persisting through 3 years of ″unrewarded″ work, in dedicated isolation, at a time when it mattered little to me that there were other mathematicians in the world besides myself, so taken was I by the fascination of what I was doing...

This unity does not derive alone as the trademark of a single worker. The themes are interconnected by innumerable ties, both subtle and obvious, as one sees in the interconnection of differing themes, each recognizable in its individuality, which unfold and develop in a grand musical counterpoint- in the harmony that assembles them together, carries them forward and assigns meaning to all of them, a movement and wholeness in which all are participants. Each of these partial themes seems to have been born out of an all-engulfing harmony and to be reborn from one instant to the next, while at the same time this harmony does not appear as a mere ″sum″ or ″resultant″ of all the themes that make it up, that in some sense are pre-existent within it. And, to speak truly, I cannot avoid the feeling (cranky as it must appear) , that in some sense it is actually this ″harmony″, not yet present but which already ″exists' somewhere in the dark womb of things awaiting birth in their time - that it is this and this alone which has inspired, each in its turn, these themes which acquire meaning only through it. And it is that harmony which called out to me in a low and impatient voice , in those solitary and inspired years of my emergence from adolescence ....

It remains true that these 12 master-themes of my work appear, as through a kind of secret predestination, to abide concurrently within the same symphony - or, to use a different image, each incarnates a different ″perspective″ on the same immense vision.

This vision did not begin to emerge from the shades, or take recognizable shape, until around the years 1957, 1958 - years of enormous personal growth. (*)

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*1957 was the year in which I began to develop the theme ″RIemann-Roch″ (Grothendieck version) - which almost overnight made me into a big ″movie star″. It was also the year of my mother's death and thereby the inception of a great break in my life story. They figure among the most intensely creative years of my entire life, not only in mathematics. I'd worked almost exclusively in mathematics for 12 years. In that year there was the sense that I'd perhaps done what there was to do in mathematics and that it was time to try something else. This came out of an interior need for revitalization, perhaps for the first time in my life. At that time I imagined that I might want to be a writer, and for a period of several months I stopped doing mathematics altogether. Finally however I decided to return, just long enough to give a definitive form to the mathematical works I'd already done, something I imagined would take only a few months, perhaps a year at most ...

The time wasn't ripe, apparently, for a complete break. What is certain is that in taking up my work in mathematics again, it took possession of me, and didn't let go of me for another 12 years!

The following year (1958) is probably the most fertile of all my years as a mathematician. This was the year which saw the birth of the two central themes of the new geometry through the launching of the theory of schemes (the subject of my paper at the International Congress of Mathematicians at Edinborough in the summer of that year) and the appearamce of the concept of a "site", a provisional technical form of the crucial notion of the topos. With a perspective of thirty years I can say now that this was the year in which the very conception of a new geometry was born in the wake of these two master-tools: schemes (metamorphosed from the anterior notion of the "algebraic variety") and the topos (a metamorphoses, even deeper, of the idea of space).

It may appear strange, but this vision is so close to me and appeared so "self-evident", that it never occured to me until about a year ago to give a name to it. (*)

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*It first occured to me to name this vision in the meditation of December 4th, 1984, (in subnote #136-1) to the footnote "Yin the Servant"(2) - or Generosity" (Récoltes et Semailles, pg. 637)

(Although it is certainly one of my passions to be constantly giving names to things that I've discovered as the best way to keep them in mind ...) It is true that I can't identify a particular moment at which this vision appeared, or which I can reconstruct through recollection. A new vision of things is something so immense that one probably can't pin it down to a specific moment, rather it takes possession of one over many years, if not over several generations of those persons who examine and contemplate it. It is as if new eyes have to be painfully fashioned from behind the eyes which, bit by bit, they are desitned to replace. And this vision is also too immense for one to speak of "grasping " it, in the same way that one "grasps" an idea that happens to arise along the way. That's no doubt why one shouldn't be surprise that the idea of giving a name to something so enormous, so close yet so diffuse, only occured to me in recollection, and then only after it had reached its full maturity.

In point of fact, for the next two years my relationship to mathematics was restricted (apart from teaching it) to just getting it done- to giving scope to a powerful impulse that ceaselesly drew me forward, into an "unknown" that I found endlessly fascinating. The idea didn't occur to me to pause, even for the space of an instant, to turn back and get an overview of the path already followed, let alone place it in the context of an evolving work. (Either for the purpose of placing it in my life, as something that continued to attach me to profound and long neglected matters; or to situate it in that collective adventure known as "Mathematics")

What must appear even more strange, in order to get me to stop for a moment and re-establish acquaintance with these half-forgotten efforts, (or to think of giving a name to the vision which is its heart and soul), I had to face a confrontation with a "Burial" of gigantic proportions: with the burial, by silence and derision, of that vision and of the worker who conceived it ...

Promenade 8

The Vision-or 12 Themes for a Harmonization

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(*)The only "semi-official" text in which these three themes are sketched, more or less, is the Outline for a Programme, edited in January 1984 on request from a unit of CNRS. This text (which is also discussed in section 3 of the Introduction, "Compass and Luggage"), should be, in principle, included in volume 4 of Mathematical Reflections.

The twelve principal themes of my opus aren't isolated from each other. To my eyes they form a unity, both in spirit and in their implications, in that one finds in them a single persistent tone, present in both "officially published" and "unpublished" writings. Indeed, even in the act of writing these lines I seem to recapture that same tone- like a call! - persisting through 3 years of "unrewarded" work, in dedicated isolation, at a time when it mattered little to me that there were other mathematicians in the world besides myself, so taken was I by the fascination of what I was doing...

This unity does not derive alone as the trademark of a single worker. The themes are interconnected by innumerable ties, both subtle and obvious, as one sees in the interconnection of differing themes, each recognizable in its individuality, which unfold and develop in a grand musical counterpoint- in the harmony that assembles them together, carries them forward and assigns meaning to all of them, a movement and wholeness in which all are participants. Each of these partial themes seems to have been born out of an all-engulfing harmony and to be reborn from one instant to the next, while at the same time this harmony does not appear as a mere "sum" or "resultant" of all the themes that make it up, that in some sense are pre-existent within it. And, to speak truly, I cannot avoid the feeling (cranky as it must appear) , that in some sense it is actually this "harmony", not yet present but which already "exists' somewhere in the dark womb of things awaiting birth in their time - that it is this and this alone which has inspired, each in its turn, these themes which acquire meaning only through it. And it is that harmony which called out to me in a low and impatient voice , in those solitary and inspired years of my emergence from adolescence ....

It remains true that these 12 master-themes of my work appear, as through a kind of secret predestination, to abide concurrently within the same symphony - or, to use a different image, each incarnates a different "perspective" on the same immense vision.

This vision did not begin to emerge from the shades, or take recognizable shape, until around the years 1957, 1958 - years of enormous personal growth. (*)

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*1957 was the year in which I began to develop the theme "RIemann-Roch" (Grothendieck version) - which almost overnight made me into a big "movie star". It was also the year of my mother's death and thereby the inception of a great break in my life story. They figure among the most intensely creative years of my entire life, not only in mathematics. I'd worked almost exclusively in mathematics for 12 years. In that year there was the sense that I'd perhaps done what there was to do in mathematics and that it was time to try something else. This came out of an interior need for revitalization, perhaps for the first time in my life. At that time I imagined that I might want to be a writer, and for a period of several months I stopped doing mathematics altogether. Finally however I decided to return, just long enough to give a definitive form to the mathematical works I'd already done, something I imagined would take only a few months, perhaps a year at most ...

The time wasn't ripe, apparently, for a complete break. What is certain is that in taking up my work in mathematics again, it took possession of me, and didn't let go of me for another 12 years!

The following year (1958) is probably the most fertile of all my years as a mathematician. This was the year which saw the birth of the two central themes of the new geometry through the launching of the theory of schemes (the subject of my paper at the International Congress of Mathematicians at Edinborough in the summer of that year) and the appearamce of the concept of a "site", a provisional technical form of the crucial notion of the topos. With a perspective of thirty years I can say now that this was the year in which the very conception of a new geometry was born in the wake of these two master-tools: schemes (metamorphosed from the anterior notion of the "algebraic variety") and the topos (a metamorphoses, even deeper, of the idea of space).

It may appear strange, but this vision is so close to me and appeared so "self-evident", that it never occured to me until about a year ago to give a name to it. (*)

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*It first occured to me to name this vision in the meditation of December 4th, 1984, (in subnote #136-1) to the footnote "Yin the Servant"(2) - or Generosity" (Récoltes et Semailles, pg. 637)

(Although it is certainly one of my passions to be constantly giving names to things that I've discovered as the best way to keep them in mind ...) It is true that I can't identify a particular moment at which this vision appeared, or which I can reconstruct through recollection. A new vision of things is something so immense that one probably can't pin it down to a specific moment, rather it takes possession of one over many years, if not over several generations of those persons who examine and contemplate it. It is as if new eyes have to be painfully fashioned from behind the eyes which, bit by bit, they are desitned to replace. And this vision is also too immense for one to speak of "grasping " it, in the same way that one "grasps" an idea that happens to arise along the way. That's no doubt why one shouldn't be surprise that the idea of giving a name to something so enormous, so close yet so diffuse, only occured to me in recollection, and then only after it had reached its full maturity.

In point of fact, for the next two years my relationship to mathematics was restricted (apart from teaching it) to just getting it done- to giving scope to a powerful impulse that ceaselesly drew me forward, into an "unknown" that I found endlessly fascinating. The idea didn't occur to me to pause, even for the space of an instant, to turn back and get an overview of the path already followed, let alone place it in the context of an evolving work. (Either for the purpose of placing it in my life, as something that continued to attach me to profound and long neglected matters; or to situate it in that collective adventure known as "Mathematics")

What must appear even more strange, in order to get me to stop for a moment and re-establish acquaintance with these half-forgotten efforts, (or to think of giving a name to the vision which is its heart and soul), I had to face a confrontation with a "Burial" of gigantic proportions: with the burial, by silence and derision, of that vision and of the worker who conceived it

Structure and Form - or the Voice of Things

Without intention on my part this "Avant-Propos" is turning, bit by bit, into a kind of formal presentation of my opus, designed above all for the non-mathematical reader. I'm too involved by now to change orientations, so I'll just plug ahead and try to bring all these "presentations" to an end! All the same, I'd like to say at least a few words on the substance of these "fabulous great ideas" (otherwise called "master themes") which I've depicted in the precedeing pagves. as well as something about the nature of this proclaimed "vision" within which these master themes are floating about. Without availaling myself of a highly technical language the most I can do is invoke the image of an intense sort of flux (if in fact one can speak of 'invoking' something)....(*)

Although this image must remain "fluid" does not mean that it isn't accurate, or that it doesn't faithfully convey something essential of the thing contemplated (in this case my opus). Conversely, it is possible to make a representation of something that is static and clear that can be highly distorted or touch only on its superficial aspects. Therefore, if you are "taken" by what I see as the essence of my work, (and something of that "image" abiding in me must have been communicated to you), you can flatter yourself to have grasped more about it than any of my learned colleagues!

It is traditional to distinguish three kinds of "qualities" or "aspects" of things in the Universe which adapt themselves to mathematical reflections. These are (1) Number(**); (2) Magnitude and (3) Form

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By this is meant the "natural numbers" o,1,2,3, etc., or (at most) the numbers (such as rational fractions) which are expressed in terms of them by the elementary operations. These numbers cannot, (as can the "real numbers") be used to measure quantities subject to continuous variation, such as the distance between two arbitrary points on a straight line, in a plane or in space.

One can also speak of them as the "arithmetical aspect", the "metric aspect" and the "geometric aspect" of things. In most of the situations studied in mathematics, these three aspects are simultaneously present in close interaction. Most often, however, one finds that one or another of them will predominate. It's my impression that for most mathematicians its quite clear to them (for those at least who are in touch with their own work) if they are "arithmeticians", "analysts", or "geometers", and this remains the case no matter how many chords they have on their violin, or if they have played at every register and diapason imaginable.

My first solitary reflections, on Measure Theory and Integration, placed me without ambiguity under the rubrique of Analysis. And this remained the same for the first of the new themes that I introduced into mathematics, (which now appears to me to be of smaller dimensions than the 11 that followed). I entered mathematics with an "analytic bias", not because of my natural temperament but owing to "fortuitous circumstances": it was because the biggest gap in my education, both at the lycée and at the university, was precisely in this area of the "analytic aspect" of things.

The year 1955 marked a critical departure in my work in mathematics: that of my passage from "analysis" to "geometry". I well recall the power of my emotional response (very subjective naturally); it was as if I'd fled the harsh arid steppes to find myself suddenly transported to a kind of "promised land" of superabundant richness, multipying out to infinity wherever I placed my hand in it, either to search or to gather... This impression, of overwhelming riches has continued to be confirmed and grow in substance and depth down to the present day.(*)

(*) The phrase "superabundant richness" has this nuance: it refers to the situation in which the impressions and sensations raised in us through encounter with something whose splendour, grandeur or beauty are out of the ordinary, are so great as to totally submerge us, to the point that the urge to express whatever we are feeling is obliterated.

That is to say that, if there is one thing in Mathematics which (no doubt this has always been so) fascinates me more than anything else, it is neither "number", nor "magnitude" but above all "form". And. among the thousand and one faces that form chooses in presenting itself to our attention, the one that has fascinated me more than any other, and continues to fascinate me, is the structure buried within mathematical objects.

One cannot invent the structure of an object. The most we can do is to patiently bring it to the light of day, with humility - in making it known it is "discovered". If there is some sort of inventiveness in this work, and if it happens that we find ourselves the maker or indefatigable builder, we aren't in any sense "making" or "building" these structures. They hardly waited for us to find them in order to exist, exactly as they are! But it is in order to express, as faithfully as possible, the things that we've been detecting or discovering, to deliver up that reticent structure, which we can only grasp at, perhaps with a language no better than babbling. Thereby are we constantly driven to invent the language most appropriate to express, with increasing refinement, the intimate structure of the mathematical object, and to "construct" with the help of this language, bit by bit, those "theories" which claim to give a fair account of what has been apprehended and seen. There is a continual coming and going, uninterrupted, between the apprehension of things, and the means of expressing them, by a language in a constant state improvement, and constantly in a process of recreation, under the pressure of immediate necessity.

As the reader must have realized by now, these "theories", "constructed out of whole cloth", are nothing less than the "stately mansions" treated in previous sections: those which we inherit from our predecessors, and those which we are led to build with our own hands, in response to the way things develop. When I refer to "inventiveness" (or imagination) of the maker and the builder, I am obliged to adjoin to that what really constitures it soul or secret nerve. It does not refer in any way to the arrogance of someone who says "This is the way I want things to be!" and ask that they attend him at his leisure, the kind of lousy architect who has all of his plans ready made in his head without having scouted the terrain, investigated the possibilities and all that is required.

The sole thing that constitutes the true "inventiveness" and imagination of the researcher is the quality of his attention as he listens to the voices of things. For nothing in the Universe speaks on its own or reveals itself just because someone is listening to it. And the most beautiful mansion, the one that best reflects the love of the true workman, is not the one that is bigger or higher than all the others. The most beautiful mansion is that which is a faithful reflection of the structure and beauty concealed within things.

The New Geometry: or the Marriage of Number and Magnitude.

But here I am, digressing again! I set out to talk about the "master-themes", with the intention of unifying them under one "mother vision", like so many rivers returning to the Ocean whose children they are..

This great unifying vision might be described as a new geometry. It appears to be similar to the one that Kronecker dreamed of a century ago(*)

(*) I only know about "Kronecker's Dream" through hearsay, in fact it was when somebody (I believe it was John Tate) told me that I was about to carry it out. In the education which I received from my elders, the historical references were very rare indeed. I was nourished, not by reading the works of others, ancient or modern, but above all through communication, through conversations or exchanges of letters, with other mathematicians, beginning with my teachers. The principal, perhaps the only external inspiration for the sudden and vigorous emergence of the theory of schemes in 1958, was the article by Serre commonly known by its label FAC (Faisceaux algébriques cohérents) that came out a few years earlier. Apart from this, my primary source of inspiration in the development of the theory flowed entirely from itself, and restored itself from one year to the next by the requirements of simplicity and internal coherence, and from my effort at taking into account in this new context, of all that was "commonly known" in algebraic geometry (which I assimilated bit by bit as it was transformed under my hands), and from all that this "knowledge" suggested to me.

But the reality is (which a bold dream may sometimes reveal, or encourage us to discover) surpasses in every respect in richness and resonance even the boldest and most profound dream. Of a certainty, for more than one of these revelations of the new geometry, (if not for all of them), nobody, the day before it appoeared, could have imagined it - neither the worker nor anyone else.

One might say that "Number" is what is appropriate for grasping the structure of "discontinuous" or "discrete" aggregates. These systems, often finite, are formed from "elements", or "objects" conceived of as isolated with respect to one another. "Magnitude" on the other hand is the quality, above all, susceptible to "continuous variation", and is most appropriate for grasping continous structures and phenomena: motion, space, varieties in all their forms, force fields, etc. Thereby , Arithmetic appears to be (overall) the science of discrete structures while Analysis is the science of continuous structures.

As for Geometry, one can say that in the two thousand years in which it has existed as a science in the modern sense of the word, it has "straddled" these two kinds of structure, "discrete" and "continuous". (*)

(*)In point of fact, it has traditionally been the "continuous" aspect of things which has been the central focus of Geometry, while those properties associated with "discreteness", notably computational and combinatorial properties, have been passed over in silence or treated as an after-thought. It was therefore all the more astonishing to me when I made the discovery, about a dozen years ago, of the combinatorial theory of the Icosahedron, even though this theory is barely scratched (and probably not even understood) in the classic treatise of Felix Klein on the Icosahedron. I see in this another significant indicator of this indifference (of over 2000 years) of geometers vis-a-vis those discrete structures which present themselves naturally in Geometry: observe that the concept of the group (notably of symmetries) appeared only in the last century (introduced by Evariste Galois), in a context that was considered to have nothing to do with Geometry. Even in our own time it is true that there are lots of algebraists who still haven't understood that Galois Theory is primarily, in essence, a geometrical vision, which was able to renew our understanding of so-called "arithmetical" phenomenon.

For some time in fact one can say that the two geometries considered to be distinct species, the discrete and the continuous, weren't really "divorced". They were rather two different ways of investigating the same class of geometric objects: one of them accentuated the "discrete" properties (notably computational and combinatorial) while the other concerned itself with the "continuous" properties (such as location in an ambient space, or the measurement of "magnitude" in terms of the distances between points, etc.)

It was at the end of the last century that a divorce became immanent, with the arrival and development of what came to be called" Abstract (Algebraic) Geometry". Roughly speaking, this consisted of introducing, for every prime number p, an algebraic geometry "of characteristic p", founded on the model (continous) of the Geometry (algebraic) inherited from previous centuries, however in a context which appeared to be resolutely "discontinuous", or "discrete". This new class of geometric objects have taken on a growing significance since the beginning of the century, in particular owing to their close connections with arithmetic, which is the science par excellence of discrete structures. This appears to be one of the notions motivating the work of André Weil (**) , perhaps the driving force (which is usually implicit or tacit in his published work, as it ought to be): the notion that "the" Geometry (algebraic), and in particular the "discrete" geometries associated with various prime numbers, ought to supply the key for a grand revitalization of Arithmetic.

(**)André Weil, a French mathematician who emigrated to the United States, is one of the founding members of the "Bourbaki Group", which is discussed in some length in the first part of Récoltes et Semailles (as is Weil himself from time to time).

It was with this perspective in mind that he announced, in 1949, his famous "Weil conjectures". These utterly astounding conjectures allowed one to envisage, for these new " discrete varieties" (or "spaces"), the possibility for certain kinds of constructions and arguments(*) which up to that moment did not appear to be conceivable outside of the framework of the only "spaces" considered worthy of attention by analysts - that is to say the so-called "topological" spaces (in which the notion of continuous variation is applicable).

One can say that the new geometry is, above all else, a synthesis between these two worlds, which, though next-door neighbors and in close solidarity, were deemed separate: the arithmeticalworld, wherein one finds the (so-called) spaces without continuity, and the world of continuous magnitudes, "spaces" in the conventional meaning of the word. In this new vision these two worlds, formerly separate, comprise but a single unity.

(*) (For the mathematical reader) The "constructions and arguments" we are refering to are associated with the Cohomology of differentiable and complex varieties, in particular those which imply the Lefschetz fixed point theorems and Hodge Theory.

The embryonic vision of this Arithmetical Geometry" (as I propose to designate the new geometry) is to be found in the Weil conjectures. In the development of some of my principal ideas(**) these conjectures were my primary source of inspiration, all through the years between 1958 and 1969.

(**)I refer to four "intermediate" themes (nos. 5 to 8) that is to say , the topos, étale and l-adic cohomology, motives and (to a lesser extent) crystals. These themes were all developed between 1958 and 1966

Even before me, in fact, Oscar Zariski on the one hand and Jean-Pierre Serre on the other had developed, for certain "wild" spaces in "abstract" Algebraic Geometry, some "topological" methods, inspired by those which had formerly been applied to the "well behaved spaces" of normal practice.(***)

(***) (For the mathematical reader) The primary contribution of Zariski in this sense seems to me to be the introduction of the "Zariski topology" (which later became an essential tool for Serre in FAC), his "principle of connectedness", and what he named the "theory of holomorphic functions" - which in his hands became the theory of formal schemes, and the theorems comparing the formal to the algebraic (with, as a secondary source of inspiration, the fundamental article by Serre known as GAGA). As for the contribution by Serre to which I've alluded in the text, it is, above all, his introduction into abstract Algebraic Geometry of the methodology of sheaves, in FAC (Faisceaux algébriques cohérents)the other fundamental paper already mentioned.

In the light of these 'reminiscences", when asked to name the immediate "ancestors" of the new geometric vision, the names that come to me right away are are Oscar Zariski, André Weil, Jean Leray and Jean-Pierre Serre.

Serre had a special role apart from all the others because of the fact that it was largely through him that I not only learned of his ideas, but also those of Zariski, Weil and Leray which were to play an important role in the emergence and development of the ideas of the new geometry.

Their ideas, without a doubt, had played an important part from my very first steps towards the building of the new geometry: furthermore, it's true, as points of departure and as tools (which I had to reshape virtually from scratch in order to adapt them to a larger context) , and a sources of inspiration which would continue to nourish my projects and dreams over the course of months and years. In any case, it's self-evident that , even in their recast state, these tools were insufficient for what was needed in making even the first steps in the direction of Weil's marvellous conjectures.

The Magical Spectrum - or Innocence

The two powerful ideas that had the most to contribute to the initiation and development of the new geometry are schemes and toposes . Having made their appearance in a somewhat symbiotic fashion at more or less the same time.

The concept of a locale or of a "Grothendieck topology" (a preliminary form of the topos) can clearly be discerned in the wake of the scheme. This, in its turn, supplies the needed new language for ideas such as "descent" and "localisation", which are employed at every stage in the development of this theme and of the schematic tools. The more inherently geometric notion of the topos, which one found only implicitely in the work of the following years, really began to define itself clearly from about 1963, with the development of étale cohomology. Bit-by-bit however it took its rightful place as the more fundamental of the two notions. To conclude this guided tour around my opus, I still need to say a few more words about these two principal ideas.

The concept of the scheme is the natural one to start with. As "self-evident" as one could imagine, it comprises in a single concept an infinite series of versions of the idea of an (algebraic) variety, that were previously used(one version for each prime number(*).

(*)It is convenient to include as well the case p = "infinity", corresponding to algebraic varieties of "nul characteristic".

In addition, one and the same "scheme" (or "variety" in the new sense) can give birth, for each prime number p, to a well-defined "algebraic variety of characteristic p". The collection of these different varieties with different characteristics can thereby be seen as a kind of" (infinite) spectrum of varieties", (one for each characteristic). The "scheme" is in fact this magical spectrum, which connects between them, as so many different "branches", its "avatars", or "incarnations" in all possible characteristics. By virtue of this it furnishes an effective "principle of transition" for tying together these "varieties", arising out of geometries which, up until that point, seemed more or less isolated, cut off from each other. For the present they are all ensconced within a common "geometry" that establishes the connections between them. One might call it Schematic Geometry, the first draft of the "Arithmetic Geometry", which was able to blossom in the coming years.

The very notion of a scheme has a childlike simplicity- so simple, so humble in fact that no one before me had the audacity to take it seriously. So "infantile" in fact, that for many years afterwards, and in spite of all the evidence, for so many of my "learned" colleagues, it was treated as a triviality. In fact I needed several months of lonely investigation to fully convince myself that the idea really "worked" - that this new language,(which, however infantile it might appear, I, in my incurable naivete continued to insist upon as something to be tested) was quite adequate for the understanding of, in a new light, with increased subtlety and in a general setting, some of the most basic geometric intuitions associated with these "geometries of characteristic p". It was a kind of exercise , prejuged by every "well informed" colleague as something idiotic and had the imagination to propose , and even (nurtured by my private demon...) follow through against all opposition!

Rather than allowing myself to be deterred by the consensus that had laid down the law over what was to be "taken seriously", and what was not, my faith was invested (as it had been in the pas) in the humble voice of phenomena, and that faculty in me which knew how to listen to it. My reward was immediate and above all expectation. In the space of only a few months, without intending to do so, I'd put my finger on several unanticipated yet very powerful tools. They've allowed me, not only to recast (as if it were play) some old results deemed difficult, in a penetrating light that went far beyond them , but also to approach and solve certain problems in "geometries of characteristic p" that until that moment had appeared inaccessible through all known methods.(*)

(*)The "proceedings" of this "forced inauguration" of the theory of schemes was the topic of my lecture at the International Congress of Mathematicians at Edinborough in 1958. The text of that talk would seem to me to be one of the best introductions to the subject from the aspect of schemes, and such as to perhaps influence a geometrician who reads it to make himself familiar, for better or worse, with the formidable treatise that followed it : Elements of Algebraic Geometry ("Eléments de Géométrie Algébrique"), which treats in a detailed (without going into technicalities!), the new foundations and the new techniques of Algebraic Geometry .

In our acquisition of knowledgeof the Universe (whether mathematical or otherwise) that which renovates the quest is nothing more nor less than complete innocence. It is in ths state of complete innocence that we receive everything from the moment of our birth. Although so often the object of our contempt and of our private fears, it is always in us. It alone can unite humility with boldness so as to allow us to penetrate to the heart of things, or allow things to enter us and taken possession of us.

This unique power is in no way a privilege given to "exceptional talents" - persons of incredible brain power (for example), who are better able to manipulate, with dexterity and ease, an enormous mass of data, ideas and specialized skills. Such gifts are undeniably valuable, and certainly worthy of envy from those who (like myself) were not so endowed at birth," far beyond the ordinary".

Yet it is not these gifts, nor the most determined ambition combined with irresistible will-power, that enables one to surmount the "invisible yet formidable boundaries " that encircle our universe. Only innocence can surmount them, which mere knowledge doesn't even take into account, in those moments when we find ourselves able to listen to things, totally and intensely absorbed in child play.

Topology - Or how to survey the fogs

The innovative notion of the "scheme", we've seen, allows one to establish connections between the different geometries associated with each prime number (or "characteristics"). These geometries, however, are all of an essentially 'discontinuous' or 'discrete' nature, as opposed to the traditional geometry which is our legacy from previous centuries, 9 back to Euclid). The new concepts introduced by Zariski and by Serre have restored, to some extent, a 'continuous dimension' concept for these geometries, which was automatically picked up by the "schematic geometry" that had just been invented to unify them. However the "fabulous conjectures" of Weil were still a long way off. These "Zariski topologies" were, seen from this perspective, so crude that one might just as well have remained at the "discrete aggregate" stage.

It was clear that what was still lacking was some new principle that could connect these geometric objects (or "varieties", or "schemes") to the usual "well behaved" (topological) "spaces": those, let us say, whose points are clearly distinguished one from the other, whearas in the "harum-scarum" spaces introduced by Zariski, the points have a sneaky tendency to cling to one another....

Most certainly it was through nothing less than through this "new principle" that the marriage of "number and magnitude", (or of "continuous and discontinuous" geometry) could give birth to the Weil conjectures.

The notion of space is certainly one of the oldest in mathematics. It is fundamental to our "geometric" perspective on the world, and has been so tacitly for over two millenia. Its only over the course of the 19th century that this concept has, bit-by-bit, freed itself from the tyranny of our immediate perceptions (that is, one and the same as the "space" that surrounds us), and of its traditional theoretical treatment (Euclidean), to attain to its present dynamism and autonomy. In our own times it has joined the ranks of those notions that are most freely and universally employed in mathematics, and is familiar, I would say, to every mathematician without exception. It has become a concept of multiple and varied aspects, of hundreds of thousands of faces depending on the kinds of structures one choses to impose on a space, from the most abundant and rich, (such as the venerable 'euclidean' structures, or the 'affine' or 'projective' ones , or again the 'algebraic' structures of similarly designated 'varieties' which generalize and extend them), down to the most 'impoverished': those in which all 'quantitative' information has been removed without a trace, or in which only a qualitative essence of "proximity" or of "limit"(*), (and, in its most elusive version, the intuition of form (called 'topological spaces')), remains.

(*)When I speak of the idea of a "limit" it is above all in terms of passage to a limit, rather than the idea that most non-mathematicians, of a "frontier"

The most "reductive" of all these notions over the course of half a century down to the present, has appropriated to itself the role of a kind of conceptual englobing substrate for all the others, that of the topological space. The study of these spaces constitutes one of the most fascinating and vital branches of geometry: Topology.

As elusive as it might appear initially, the "qualitatively pure" structure encapsulated in the notion of "space"(topological) in the absence of all quantitative givens, (notably the metric distances between points) which enables us to relate it to habitual intuitions of "large" and "small", we have, all the same, over the last century, been able to confine these spaces in the locked flexible suitcases of a language which has been meticulously fabricated as the occasion arose. Still better, as the occasion arose, various 'weights and measures' have been devised to serve a general function, good or bad, of attaching "measures" (called 'topoligical invariants'), to those sprawled-out spaces which appear to resist, like fleeting mists, any sort of metrizability. Most of these invariants, its true, certainly the most essential ones, are more subtle than simple notions like 'number' and 'magnitude' - often they are themselves fairly delicate mathematical structures bound (by rather sophisticated constructions) to the space in question. One of the oldest and most crucial of these invariants, introduced in the last century (by the Italian mathematician Betti) is formed from the various "groups" (or 'spaces'), called the "Cohomology" associated with this space. (*)

(*) Properly speaking, the Betti invariants were homological invariants. Cohomology is a more or less equivalent or "dual" version that was introduced much later. This has gained pre-eminence over the intial "homological" aspect, doubtless as a consequence of the introduction, by Jean Leray, of the viewpoint of sheaves, which is discussed further on. From the technical point of view one can say that a good part of my work in geometry has been to identify and develop at some length, the cohomological theories which were needed for spaces and varieties of every sort, above all for the "algebraic varieties" and the schemes. Along the way I was also led to a reinterpretation of the traditional homological invariants in cohomological terms, and through doing so, to reveal them in an entirely new light.

There are numerous other "topological invariants" which have been introduced by the topologists to deal with this or that property or this or that topological space. Next after the "dimension" of a space and the (co)homological invariants, come the "homotopy groups". In 1957 I introduced yet another one, the group (known as "Grothendieck) K(X), which has known a sensational success and whose importance (both in topology and arithmetic) is constantly being re-affirmed. A whole slew of new invariants, more sophisticated than the ones presently known and in use, yet which I believe to be fundamental, have been predicted by my "moderated topology" program (one can find a very summary sketch of this in the "Outline for a Programme" which appears in Volume 4 of the Mathematical Reflections) . This programme bases itself on the notion of a "moderated theory" or "moderated space", which constitutes, a bit like the topos, a second "metamorphosis of the concept of space". It is at the same time more self-evident and less prodound than the latter. I predict that its immediate applications to topology "properly speaking" will be decidedly more incisive, that in fact it will turn upside down the "profession" of topological geometer, through a far-reaching transformation of the conceptual context appropriate to it. (As was the case with Algebraic Geometry with the introduction of the point-of-view of the scheme.) Furthermore, I've already sent copies of my "Outline" to several of my old friends and some illustrious topologists, yet it seems to me that that haven't been inclined to take any interest in it....

It was the Betti numbers that figure ("between the lines" naturally) in the Weil conjectures, which are their fundamental "reason for being" and which (at least for me, having been "let in on the secret" by Serre's explications) give them meaning. Yet the possibility of associating these invariants with the "abstract" algebraic varieties that enter into these conjectures, in such a manner as to response to the very precise desiderata demanded by the requirements of this particular cause - that was something only to be hoped for. I doubt very much that, outside of Serre and myself, there isn't anyone else (including André Weil!)who really believes in it.(*)

(*) It is somewhat paradoxical that Weil should have an obstinate, even visceral block against the formalism of cohomology, particularly since it had been in large part his "famous" conjectures that inspired the development, starting in 1955, of the great cohomological theories of algebraic geometry, (launched by J.P. Serre with his foundational article "FAC", already alluded to in a footnote.)

Its my opinion that this "block" is part of a general aversion in Weil against all the global formalisms, (whether large or small), or any sort of theoretical construction. He hasn't anything of the true "builder" 蔭bout him, and it was entirely contrary to his personal style that he saw himself constrained to develop, starting with the 30's, the fundamentals of "abstract" algebraic geometry, which to him (by his own dispositions), have proved to be a veritable "Procrustean bed" for those who use them.

I hope he doesn't hold it against me that I chose to go beyond him, investing my energy in the construction of enormous dwelling places, which have allowed the dreams of a Kronecker, and even of himself, to be cast into a language and tools that are at the same time effective and sophisticated. At no time did he ever comment to me about the work that he saw me doing, or which had already been done. Nor have I received any response from him about Récoltes et Semailles, which i sent to him over three months ago, with a warm hand-written personal dedication to him.

Soon afterwards our understanding of these cohomological invariants was profoundly enriched and renovated by the work of Jean Leray (carried out as a prisoner of war in Germany in the early part of the 40's). The essential novelty in his ideas was that of the (Abelian) sheaf over a space, to which Leray associated a corresponding collection of cohomology groups (called "sheaf coefficients"). It is as if the good old standard "cohomological metric" which had been used up to them to "measure" a space, had suddenly multiplied into an unimaginablely large number of new "metrics" of every shape, size and form imaginable, each intimately adapted to the space in question, each supplying us with very precise information which it alone can provide. This was the dominant concept involved in the profound transformation of our approach to spaces of every sort, and unquestionably one of the most important mathematical ideas of the 20th century.

Thanks above all to the ulterior work of Jean-Pierre Serre, Leray's ideas have produced in the half century since their formulation, a major redirection of the whole theory of topological spaces, (notably those invariants designated as "homotopic", which are intimately allied with cohomology), and a further redirection, no less significant, of so-called "abstract"薩lgebraic geometry (starting with the FAC article of Serre in 1955). My early work in geometry, from 1955 onwards, was conceived of as a continuation of the work of Serre, and for that reason also a continuation of the work of Leray.

Toposes or The Double Bed

The new perspective and language introduced by the use of Leray's concepts of sheaves has led us to consider every kind of "space" and "variety" in a new light. These did not however have anything to say about the concept of space itself, and was content if it enabled us to refine our understanding of the already traditional and familiar "spaces". At the same time it was recognized that this way of looking at space was insufficient for taking into account the "topological invariants" which were most essential for expression the "form" of these "abstract algebraic varieties" (such as those which figure in the Weil Conjectures), let alone that of general "schemes" (for the most part the classical varieties). For the desired "marriage" of "Number and Magnitude" one would have a rather narrow bed, one in which at most one of the future spouses (for example, the bride) could accomodate herself for better or worse, but never both at the same time! The "new principle" that needed to be found so that the marriage announced by the guardian spirits could be consummated, was simply that missing spacious bed, though nobody at the time suspected it.

This "double bed" arrived (as from the wave of a magic wand) with the idea of the topos. This idea encapsulates, in a single topological intuition, both the traditional topological spaces, incarnation of the world of the continuous quantity, and the so-called "spaces" (or "varieties") of the unrepentant abstract algebraic geometers, and a huge number of other sorts of structures which until that moment had appeared to belong irrevocably to the "arithmetic world" of "discontinuous" or "discrete" aggregates.

It was certainly the sheaf perspective that was my sure and quiet guide, the right key (hardly secret) to lead me without detours nor procrastination towards the nuptial chamber and its vast conjugal bed. A bed so enormous in fact (like a vast, deep and peaceful stream) in which

"Tous Les Chevaux du Roi

Y pourraient boire ensemble"

- as the old ballad that you must surely have heard or sung at one point tells us.

And he who was the first to sing it was he who has best savored the secret beauty and passive force of the topos, better than any of my clever students and former friends ....

It was the same key, both in the initial and provisional approach via the convenient, yet unintrinsic, concept of a "site" , as with the topos. I will now attempt to describe the topos concept.

Consider the set formed by all sheaves over a (given) topological space or, if you like, the formidable arsenal of all the "rulers" that can be used in taking measurements on it. (*)

(*) (For the mathematician): properly speaking, one is speaking of sheaves of ensembles, not the Abelian sheaves introduced by Leray as generalized coefficients in the formation of "cohomology groups" I believe that I'm the first person to have worked systematically with sheaves of ensembles (starting in 1955 at the University of Kansas, with my article "A general theory of fibre spaces with structure sheaf")

We will treat this "ensemble", or "arsenal" as one equipped with a structure that may be considered "self-evident", one that crops up "in front of one's nose": that is to say, a Categorical structure. (Let not the non-mathematical reader trouble himself if he's unaware of the technical meanings of these terms, which will not be needed for what follows).

It functions as a kind of "superstructure of measurement", called the "Category of Sheaves" (over the given space), which henceforth shall be taken to incoorporate all that is most essential about that space. This is in all respects a lawful procedure, (in terms of "mathematical common sense") because it turns out that one can "reconstitute" in all respects, the topological space(**) by means of the associated "category of sheaves" (or "arsenal" of measuring instruments)

(**) (For the mathematical reader) Strictly speaking, this is only true for so-called "tame" spaces. However these include virtually all of the spaces one has to deal with, notably the "separable spaces" so dear to functional analysts.

(The verification of this is a simple exercise- once someone thinks to pose the question, naturally) One needs nothing more (if one feels the need for one reason or another), henceforth one can drop the initial space and only hold onto its associated "category" (or its "arsenal"), which ought to be considered as the most complete incarnation of the "topological (or spatial) structure" which it exemplifies

As is often the case in mathematics, we've succeeded (thanks to the crucial notion of a "sheaf" or "cohomological ruler") to express a certain idea (that of a "space" in this instance), in terms of another one (that of the "category"). Each time the discovery of such a translation from one notion (representing one kind of situation) to another (which corresponds to a different situation) enriches our understanding of both notions, owing to the unanticipated confluence of specific intuitions which relate first to one then to the other. Thus we see that a situation said to have a "topological" character (embodied in some given space) has been translated into a situation whose character is "algebraic" (embodied in the category); or, if you wish, "continuity" (as present in the space) finds itself "translated" or "expressed" by a categorical structure of an "algebraic" character, (which until then had been understood only in terms of something "discrete" or "discontinuous".)

Yet there is more here. The first idea, that of the space, was perceived by us as a "maximal" thing - a notion already so general that one could hardly envisage any kind of "rational" extension to it. On the contrary, it has turned out that, on the other side of the mirror(*)

(*) The"mirror" refered to, as in Alice in Wonderland, is that which yields as the "image" of a space placed in front of it, the associated "category", considered as a kind of "double" of the space , on the "other side of the mirror(*)

these "categories", (or "arsenals") one ends up with in dealing with topological spaces, are of a very particular character. Their collection of traits is in fact highly specific.(**), and tend to join up in patchwork combinations of an unbelievably simple nature- those which on can obtain by taking as one's point of departure the reduction of a space to a single point.

(**) (For the mathematical reader) We're speaking about primarily the properties which I introduced into Category Theory under the name of "exact characteristics", (along with the categorical notions of general projective and inductive limits). See " On several points of homological algebra", Tohoku Math Journal, 1957 (p. 119-221))

Having said this, a "space defined in the new way" (or topos) one that generalizes the traditional topological space, can be simply described as a "category" which, without necessarily deriving from an ordinary space, nevertheless possesses all of the good properties (explicitely designated once and for all, naturally) of the "sheaf category".

This therefore is the new idea. Its appearance may perhaps be understood in the light of the observation, a childlike one at that, that what really counts in a topological space is neither its "points" nor its subsets of points.(*), nor the proximity relations between them, rather it is the sheaves on that space, and the category that they produce.

(*)Thus, one can actually construct "enormous" topoi with only a single point, or without any points at all!

All that I've done was to draw out the ultimate consequences of the initial notion of Leray - and by doing so, lead the way .

As even the idea of sheaves (due to Leray), or that of schemes, as with all grand ideas that overthrow the established vision of things, the idea of the topos had everything one could hope to cause a disturbance, primarily through its "self-evident" naturalness, through its simplicity (at the limit naive, simple-minded, "infantile")- through that special quality which so often makes us cry out: "Oh, that's all there is to it!", in a tone mixing betrayal with envy, that innuendo of the "extravagant", the "frivolous", that one reserves for all things that are unsettling by their unforseen simplicity, causing us to recall, perhaps, the long buried days of our infancy....

Mutability of the Concept of Space-or Breath and Faith

The notion of the scheme constitutes a great enrichment of the notion of the "algebraic variety". By virtue of that fact it has successful renovated, from top to bottom, the subject of Algebraic Geometry left to me by my predecessors. The notion of the topos however constitutes an altogether unsuspected extension, more accurately a metamorphoses of the concept of space . Thereby it holds the promise to effect a similar renovation of the subject of Topology and, beyond that, Geometry. Furthermore, at present it has already played a crucial role in the growth and development of the new geometry (above all by means of the methods of p-adic and crystalline cohomology which have come out of it and, thereby, the proofs of the Weil conjectures.) As its elder sister (quasi twin) it contains the pair of complementary characteristics essential to every fertile generalization, to wit:

Primo , the new concept isn't too large, in the sense that within these new "spaces", (or, for the sake of overly delicate ears [1], "toposes") the most essential "geometric" intuitions [2] and constructions, familiar to us from the old traditional spaces, can be easily transposed in an evident manner. In other words, one has at one's disposal in these new objects the rich collection of images and mental associations, of ideas and certainly some techniques, that were formerly confined to objects of the earlier sort.

Secundo, the new concept is large enough to encapsulate a host of situations which, until now, were not considered capable of supporting intuitions of a "topologic-geometric" nature - those intuitions, indeed, which had been reserved in the past exclusively for the ordinary topological spaces (and for good reason....)

What is crucial, from the standpoint of the Weil conjectures, is that the new ideas be ample enough to allow us to associate with every scheme such a "generalized space" or "topos" (called the "étale topos" of the corresponding scheme) . Certain "cohomological invariants" of this topos (nothing can be more "childishly simple" !) then appeared to furnish one with "what was needed" in order to bring out the full meaning of these conjectures, and perhaps (who knew then!) supply the means for demonstrating them.

It's in the pages that I'm in the process of writing at this very moment that , for the first time in my life as a mathematician, I can take the time needed to evoke (if only for myself) the ensemble of the master-themes and motivating ideas of my mathematical work. It's lead me to an appreciation of the role and the extensions of each of these themes and the "viewpoints" they incarnate, in the great geometric vision that unite them and from which they've issued. It is through this work that the two innovative ideas of the first powerful surge of the new geometry first saw the light of day: that of schemes and that of the topos .

It's the second of these ideas, that of the topos, which at this moment impresses me as the more profound of the two of them. Given that I, at the end of the 50's, rolled up my sleeves to do the obstinate work of developing , through twelve long years, of a "schematic tool" of extraordinary power and delicacy, it is almost incomprehensible to me that in the ten or twenty years that have since followed, others besides myself have not carried through the obvious implications of these ideas, or raised up at least a few dilapidated "prefabricated" shacks as a contribution to the spacious and comfortable mansions that I had the heart to build up brick by brick and with my own bare hands.

At the same time, I haven't seen anyone else on the mathematical scene, over the last three decades, who possesses that quality of naivete, or innocence, to take (in my place) that crucial step, the introduction of the virtually infantile notion of the topos, (or even that of the "site"). And, granted that this idea had already been introduced by myself, and with it the timid promise that it appeared to hold out - I know of no-one else, whether among my former friends or among my students, who would have had the "wind", and above all the "faith", to carry this lowly notion [3] to term (so insignificant at first sight, given that the ultimate goal appeared infinitely distant...) : since its first stumbling steps, all the way to full maturity of the "mastery of étale cohomology", which, in my hands, it came to incarnate over the years that followed.

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[1] Nomenclature: the name "topos" was chosen (with its associations to "topology" and "topological"), to imply that it was the "principal object" to which "topological intuition" inheres. Through the rich cloud of mental images that this name evokes, one ought to consider it as more or less equivalent to the term "space" (topological), with the requirement that the notion of the "topological" be more precisely specified. (In the same

way that one has "vectorial spaces", but on "vectorial toposes", at least for the moment!) It's important to maintain both expressions together, each with its proper specificity.

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[2] Among these "constructions" one finds the familiar "topological invariants", including the cohomological invariants. For these I've done all that's necessary in the article previously cited ("Tohoku" 1955) in order to give them a proper meaning for each "topos".

[3] (For the mathematical reader) When I speak of "wind" and of "faith", I'm referring to characteristics of a non-technical nature , although I consider them to be essentially necessary characteristics. At another level I might add that I have referred to the "cohomological flair", that is to say the sort of aptitude that was developed in me through the erection of theories of cohomology. I believed that I was able to transmit this to my students in cohomology. With a perspective of 17 years after my departure from the world of mathematics, I can say that not a one of them had developed it.

"Tous les Chevaux du Roi..."

Verily, the river is deep, and peaceful and vast are the waters of my infancy, in a kingdom which I'd believed to have left so long ago. All the king's horses may come and drink at their leisure, quenching of their thirst without the waters ever drying up! They descend from the glaciers, full of the ardor of distant snows, with the sweetness of the clay of the plains. I've just written about one of those horses, which was led to drink by a child and which drank at length to its full content. And I saw another that came to drink for a moment or two, in search of that same youngster - but it did not linger. Someone must have chased it off. And, to speak truly, that's all. Yet I also see numberless herds of horses who wander the plains, dying of thirst - as recently as this morning their whinnying dragged me from my bed, and at an unaccustomed hour, although I am on the verge of my 60's and cherish my tranquillity. There was no help for it, I was obliged to get up. It gave me pain to see them, horridly raw-boned and skinny, although there was no lack of abundance of good water or green pasture. Yet one might speak of a kind of malignant magical spell that has fallen over the land that I once found so accomodating, contaminating its generous waters. Who knows? One could imagine that some kind of plot had been hatched by the horse-traders of the land to bring down prices! Or it may be that this country no longer possesses any children for leading the horses to water, and that the horses will remain thirsty until there is a child who rediscovers the road that leads to the stream....

The "topos" theme came from that of "schemes" in the year of their appearance; yet it has greatly surpassed the mother notion in its extent. It is the topos, not schemes, which is the "bed", or that "deep river", in which the marriage of geometry, topology and arithmetic, mathematical logic, the theory of categories, and that of continous and discontinuous or "discrete" structures, is celebrated. If the theme of schemes is at the heart of the new geometry, the theme of the topos envelopes it as a kind of residence. It is my grandest conception, devised in order to grasp with precision, in the same language rich in resonances of geometry, an "essence" common to the most disparate situations, coming from every region of the universe of mathematical objects.

Yet the topos has not known the good fortune of the schemes. I discuss this subject in several places in Récoltes et Semailles, and this is not the place at which to dwell upon the strange adventures which have befallen this concept. However, two of the principal themes of the new geometry have derived from that of the topos, two "cohomological theories" have been conceived, one after the other, with the same purpose of providing an approach to the Weil conjectures: the étale (or l-adic) theme , and the crystalline theme.

The first was given concrete form in my hands as the tool of l-adic cohomology, which has been shown to be one of the most powerful mathematical tools of this century.

As for the crystalline theme, (which had been reduced since my departure to a virtually quasi-occult standing) , it has finally been revitalized (under the pressure of necessity), in the footlights and under a borrowed name, in circumstances even more bizarre than those which have surrounded the topos.

As predicted, it was the tool of l-adic cohomology which was needed to solve the Weil conjectures. I did most of the work, before the remainder was accomplished, in a magistral fashion, 3 years after my departure, by Pierre Deligne, the most brilliant of all my "cohomological" students.

Around 1968 I came up with a stronger version, (more geometric above all), of the Weil conjectures. These are still "stained" (if one may use that expression) with an "arithmetical" quality which appears to be irreducible. All the same, the spirit of these conjectures is to grasp and express the "arithmetical" (or discrete) through the mediation of the "geometric" (or the "continuous".)(*)

(*For the mathematical reader) The Weil conjectures are subject to hypotheses of an essentially arithmetical nature, principally because the varieties involved must be defined over finite fields. From the point of view of the cohomological formalism, this results in a privileged status being ascribed to the Frobenius endomorphism allied with such situations. In my approach, the crucial properties (analogous to 'generalized index theorems') are present in the various algebraic correspondances, without making any arithmetic hypotheses about some previously assigned field.

In this sense the version of these conjectures which I've extracted from them appears to my mind to be more "faithful" to the "Weil philosophy" than those of Weil himself! - a philosophy that has never been written down and rarely expressed, yet which probably has been the primary motivating force in the extraordinary growth and development of geometry over the course of the last 4 decades.(*)

(*)Since my departure in 1970 however, a reactionary tendency has set in, finding its concrete expression in a state of relative stagnation, which I speak of on several occasions in the pages of Récoltes et Semailles.

My reformulation consisted, essentially, in extracting a sort of "quintessence" of what is truly valuable in the framework of what are called "abstract" algebraic varieties, in classical "Hodge theory", and in the study of "ordinary" algebraic varieties.(*)

(*)Here the word 'ordinary' signifies:"defined over complex fields". Hodge theory (for "harmonic integrals") was the most powerful of the known cohomological theories in the context of complex algebraic varieties.

I've named this entirely geometric form of these celebrated conjectures the " standard conjectures".

To my way of thinking, this was, after the development of l-adic cohomology, a new step in the direction of these conjectures. Yet, at the same time and above all, it was also one of the principal possible approaches towards what still appears to me to be the most profound of all the themes I've introduced into mathematics (*) , that of motives, (themselves originating in the "l-adic cohomology theme")

(*)This was the deepest theme at least during my period of mathematical activity between 1950 and 1969, that is to say up to the very moment of my departure from the mathematical scene. I deem the themes of anabelian algebraic geometry and that of Galois-Teichmuller theory, which have developed since 1977, to be of comparable depth.

This theme is like the heart, or soul, that which is most hidden, most completely shielded from view within the "schematic" theme, which is itself at the very heart of the new vision. And several key phenomena retrieved from the standard conjectures (**) can also be seen as constituting a sort of ultimate quintessence of the motivic theme, like the "vital breath" of this most subtle of all themes, of this "heart within the heart" of the new geometry.

(**) (For the algebraic geometer). Sooner or later there must be a revision of these conjectures. Fir more detailed commentary, go to "The tower of scaffoldings" (R&S IV footnote #178, p. 1215-1216), and the note at the bottom of page 769, in "Conviction and knowledge" (R&S III, footnote#162)

Roughly speaking, this is what's involved. We've come to understand , for a given prime number p, the importance of knowing how to construct "cohomological theories" (particularly in light of the Weil conjectures) for the "algebraic varieties of characteristic p" . Now, the celebrated "cohomological l-adic tool" supplies one with just such a theory, and indeed, an infinitude of different cohomological theories , that is to say, one associated with each prime number different from p. Clearly there is a "missing" theory, namely that in which l and p are equal. In order to provide for this case I conceived of yet another cohomological theory (to which I've already alluded), entitled "crystalline cohomology".

Furthermore, in the case in which p is infinite, there are yet 3 more cohomological theories (***)

(***) (For the benefit of the mathematical reader) These theories correspond, respectively, toBetti cohomology(by means of transcendentals, and with the help of an embedding of the base field into the field of the complex numbers), Hodge cohomology, and de Rham cohomology as interpreted by myself. The latter two date back to the 50's (that of Betti to the 19th century).

Furthermore there is nothing to prevent the appearance, sooner or later, of yet more cohomological theories, with totally analogous formal properties. In contradistinction to what one finds in ordinary topology, one finds oneself in the presence of a disconcerting abundance of differing cohomological theories. One had the impression that, in a sense that should be taken rather flexibly, all of these theories "boiled down" to the same one, that they "gave the same results". (****)

(****)(For the benefit of the mathematical reader). For example, if f is an endomorphism of the algebraic variety X, inducing an endomorphism of the cohomology space Hi(X), then the fact that the "characteristic polynomial" of the latter must have integrer coefficients does not depend on the kind of cohomology employed (for example, l-adic for some arbitrary l) . Likewise for algebraic correspondances in general, which X is presumed proper and smooth. The sad truth, (and this gives one an idea of the deporable state in which the cohomological theory of algebraic varieties of characterstic p finds itself since my departure), is that there is no demonstration of this fact, as of this writing, even in the simplest case in which X is a smooth projective surface , and i = 2. Indeed, to my knowledge, nobody since my departure has deigned to interest himself in this crucial question, which is typical of all those which are subsidiary to the standard conjecture. The doctrine a-la-mode is that the only endomorphism worthy of anyone's attention is the Frobenius endomorphism, (which could have been treated by Deligne by the method of boundaries ...)

It was through my intention to give expression to this "kinship" between differing cohomological theories that I arrived at the notion of associating an algebraic variety with a "motive". My intention in using this term is to suggest the notion of the "common motive" (or of the "common rationale") subsidiary to the great diversity of cohomological invariants associated with the variety, owing to the enormous collection of cohomologies possible apriori .The differing cohomological theories would then be merely so many differing thematic developments, (each in the "tempo" , the "key", and "mode" ("major" or "minor") appropriate to it) , of an identical "basic motive" (called the "motivic cohmological theory"), which would also be at the same time the most fundamental, the ultimate "refinement" of all the differing thematic incarnations (that is to say, of all the possible cohomological theories).

Thus the motive associated with an algebraic variety would constitute the ultimate invariant, the invariant par excellence from the cohomological standpoint among so many musical "incarnations", or differing "realizations". All of the essential properties of the cohomology of the variety could already be read off (or be "extended to") on the corresponding motive, with the result that the properties and familiar structures of particular cohomological invariants, (l-adic, crystalline for example) would be merely the faithful reflection of the properties and structures intrinsic to the motive(*) .

(*)(For the benefit of the mathematical reader). Another way of viewing the category of motives over a field k, is to visualize it as a kind of "covering Abelian category" of the category of distinct schemes of finite type over k. Then the motive associated with a given schema X ("cohomological motive" of X which I notate as H*(mot)(X)) thereby appears as a sort of "Abelianized avatar" of X. The essential point is that , even as an Abelian variety X is susceptible to "continuous variation" (with a dependence of its' isomorphism class on "continuous parameters", or "modules") , the motive associated with X , or more generally, a "variable" motive, is also susceptible to continuous variation.This is an aspect of motivic cohomology which is in flagrant contrast to what one normally has with respect to all the classical cohomological invariants, (including the l-adic invariants), with the sole exception of the Hodge cohomology of complex algebraic varieties.

This should give one an idea of to what extent "motivic cohomology" is a more refined invariant, encapsulating in a far tighter manner the "arithmetical form" (if I can risk such an expression) of X, than do the traditional invariants of pure topology. In my way of looking at motives, they consitute a kind of delicate and hidden "thread" linking the algebraic-geometric properties of an algebraic variety to the properties of an "arithmetic" nature incarnated in its motive. The latter may then be considered to be an object which, in its spirit, is geometric in nature, yet for which the "arithmetic" properties implicit in its geometry have been laid bare.

Thus, the motive presents itself as the deepest "form invariant" which one has been able to associate up to the present moment with an algebraic variety, setting aside its "motivic fundamental group". For me both invariants represent the "shadows" projected by a "motivic homotopy type" which remains to be discovered (and about which I say a few things in the footnote: "The tower of scaffoldings- or tools and vision" (R&S IV, #178, see scafolding 5 (Motives), and in particular page 1214)) .

It is the latter object which appears to me to be the most perfect incarnation of the elusive intuition of "arithmetic form" (or "motivic"), of an arbitrary algebraic variety.

Here we find, expressed in the untechnical language of musical metaphor, the quintessence of an idea (both delicate and audacious at once), of virtually infantile simplicity. This idea was developed, on the fringes of more fundamental and urgent tasks, under the name of the "theory of motives", or of "philosophy (or "yoga") of the "motives", through the years 19673-69. It's a theory of a fascinating structural richness, a large part of which remains purely conjectural. (*)

(*)I've explained my vision of motives to any who wished to learn about them all through the years, without taking the trouble to publish anything in black and white on this subject (not lacking in other tasks of importance). This enabled several of my students later on to pillage me all the more easily, and under the tender gaze of my circle of friends who were well aware of the situation. (See the following footnote)

IN R &S I often return to this topic of the "yoga of motives", of which I am particularly fond. There is no need to dwell here on what is discussed so throughly elsewhere. It suffices for me to say that the "standard conjectures" flow in a very natural way from the world of this yoga of motives. These conjectures furnish at the same time a primary means for effecting one of the possible formal constructions of the notion of the motive.

The standard conjectures appeared to me then, and still do today, as one of the two questions which are the most fundamental in Algebraic Geometry. Neither this question, nor the other one (known as the "resolution of singularities") has been answered at the present time. However, whereas the second of them has a venerable history of a century, the other one, which I've had the honor of discovering, now tends to be classfied according to the dictates of fad-and-fashion (over the years following my departure from the mathematical scene, (and similarly for the theme of motives)) , as some kind of genial "grothendieckean" fol-de-rol. Once more I'm getting ahead of myself .... (*)

(*) In point of fact, this theme was exhumed (one year after the crystalline theme) , but this time under its own name , (and in a truncated form, and only in the single case of a base field of null characteristic) , without the name of its discoverer being so much as mentioned. It constiutes one example among so many others, of an idea and a theme which were buried at the time of my departure as some kind of "grothendieckean" fantasmagoria", only to be revived, one after another, by certain of my students over the course of the next 10 to 15 years, with shameless pride and (need one spell it out?) never a mention of its originator.

In Quest of the Mother - Two Views

Speaking truthfully, my thoughts about the Weil conjectures in and of themselves, that is to say with the goal of solving them, have been sporadic.The panorama that opened up before me, which I was obliged to make the effort to scrutinize and capture, greatly surpassed in scope and in depth the hypothetical needs for proving these conjectures, or indeed all the results that would follow from them. With the emergence of the themes of the schemes and topos, an unsuspected world suddenly opened up. Certainly the "conjectures" occupy a central place, in much the way as the capital city of a vast empire or continent, with numberless provinces, most of which have only the most tenuous relations with the brilliant and prestigious metropolis. Without having to make it explicit, I knew that henceforth I was to be the servant of a great enterprise: to explore this immense and unknown world, to depict its frontiers however far distant: to traverse it in all directions, to inventory with obstinate care the closest and most accessible of these provinces; then to draw up precise maps in which the least little village and tiniest cottage would have their proper place ...

It is the later task, above all, which absorbed most of my energy - a long and patient labor on foundations, which I was the first to see with clarity and, above all, to "know in my guts". It is this which took up the major part of my time between 1958 (the year in which one after another, the schemes and the topos made their respective appearances), and 1970, (the year of my departure from the mathematical scene.)

It often happened also that I chaffed at the bit to be constrained in this fashion, like someone pinned down by an immovable weight, by those interminable tasks which (once the essentials had been understood) seemed more of a routine character than a setting forth into the unknown. I had constantly to restrain the impulse to thrust forward - in the manner of a pioneer or explorer, occupied somewhere far distant in the discovery and exploration of unknown and nameless worlds, crying out for me to become acquainted with them and bestow names upon them. This impulse, and the energy I invested in them , (partially, ,in my spare time), were constantly held in abeyance.

However I knew very well that it was this energy, so slight, (in a manner of speaking)in comparison with what I gave to my "duties", that was the most important and advanced; in my "creative" work in mathematics it was this that was involved; in that intense attention given to the apprehension of, in the obscure folds, formless and moist, of a hot and inexhaustibly nourishing womb, the earliest traces and shapes of what had yet to be born and which appeared to be calling out to me to give it form, incarnation and birth ... This work of discovery, the concentrated attention involved, and its ardent solicitude, constituted a primeval force, analogous to the sun's heat in the germination and gestation of seeds sown in the nourishing earth, and for their miraculous bursting forth into the light of day.

In my work as a mathematician I've seen two primary forces or tendancies of equal importance at work, yet of totally different natures- or so it seems to me. To evoke them I've made use of the images of the builder, and of the pioneer or explorer. Put alongside each other, both strike me somehow as really quite "yang", very "masculine", even "macho"! They possess the heightened resonance of mythology, of "great events". Undoubtably they've been inspired by the vestiges within me of my old "heroic" vision of the creative worker, the "super-yang" vision. Be that as it may, they produce a highly colored image, if not totally pictorial yet "standing at attention" to be viewed, of a far more fluid, humble and "simple" reality -one that is truly living.< P> However, in this "male" "builder's" drive, which would seem to push me relentlessly to engineer new constructions I have, at the same time, discerned in me something of the homebody, someone with a profound attachment to " the home". Above all else, it is "his" home, that of persons "closest" to him- the site of an intimate living entity of which he feels himself a part. Only then, and to the degree which the circle of his "close associates" can be enlarged, can it also be a "open house" for everyone.

And, in this drive to "make" houses (as one "makes" love...) there is above all, tenderness. There is furthermore the urge for contact with those materials that one shapes a bit at a time, with loving care, and which one only knows through that loving contact. Then, once the walls have been erected, pillars and roof put in place, there comes the intense satisfaction of installing the rooms, one after the other, and witnessing the emergence, little by little, from these halls, rooms and alcoves, of the harmonious order of a living habitation - charming, welcoming, good to live in. Because the home, above all and secretly in all of us, is the Mother - that which surrounds and shelters us, source at once of refuge and comfort; and it is even (at a still deeper level, and even as we are in the process of putting it all in place), that place from which we are all issued, which has housed and nourished us in that unforgettable time before our birth... It is thus also the Busom.

And the other spontaneously generated image, going beyond the inflated notion of a "pioneer', and in order to grasp the hidden reality which it conceals, is itself devoid of all sense of the "heroic". There once again, it is the archetypal maternal image which occurs - that of the nourishing "matrix" , and of its formless and obscure labors...

These twin urges which appeared to me as being "totally different" have turned out to be much closer than I would have imagined. Both the one and the other have the character of a "drive for contact", carrying us to the encounter with "the Mother": that which incarnates both that which is close and "known", and that which is "unknown". In abandoning myself to either one or the other, it is to "rediscover the Mother", it is in order to renovate contact with that which is near, and "more or less known", and that which is distant, yet at the same time sensed as being on the verge of being understood.

The distinction is primarily one of tone, of quantity, but not of anessential nature. When I "construct houses", it is the "known" which dominates; when I "explore", it is the "unknown". These two "modes" of discovery, or to better state the matter, these two aspects of a single process, are indissolubly linked. Each is essential and complementary to the other. In my mathematical work I've discerned a coming-and-going between these two ways of approaching things, or rather, between those moments (or periods) in which one predominates, then the other (*)

(*) What I've been saying about mathematical work is equally true for "meditative" activity (which is discussed more or less throughout Récoltes et Semailles). I have no doubts that it is innate to all forms of discovery, including those of the artist (writer or poet for example). The two "faces" which I've described here might also be seen as being, on the one hand that of expression and its "technical" requirements, while the other is that of reception (of perceptions and impressions of all sorts), turning into inspiration as a consequence of intense concentration. Both the one and the other are present at every working moment, as well as that 'coming-and-going', in which first the one predominates, then the other.

Yet it is also clear that , at every instant, one or the other mode will be present. When I construct, furnish, clear out the rubble or clean the premises, or set things in order, it is the "mode", or "face" of the "yang", the "masculine" which sets the tone of my work. When I explore, groping around that which is uncomprehended, formless, that which is yet without any name, I'm following the "yin" aspect, or "feminine" side of my being.

I've no intention of wishing to minimize or denigrate either side of my nature, each essential one to the other: the "masculine" which builds and engenders, or the "feminine" which conceives, which shelters the long and obscure pregnancies. I "am" either one or the other - "yang" and "yin", "man" and "woman". Yet I'm also aware that the more delicate, the subtler in unravelling of these creative processes is to be found in the "yin" or "feminine" aspect - humble, obscure, often mediocre in appearance.

It's this side of my labor which, always I would say, has held the greatest fascination for me. The modern consensus however had tried to encourage me to invest the better part of my energy in the other side, in those efforts which affirm themselves by being incarnated in "tangible" products, if not always finished or perfected - products with well-defined boundaries, asserting their reality as if they'd been cut in stone... I can now see, upon reflection, how heavily this consensus weighed on me, and also how I "bore the weight of the accusation"-with submission! The aspect of "conception", or "exploration" of my work was accorded a meagre role by me, even up to the moment of my departure. And yet, in the retrospective overview I've made of my work as a mathematician, the evidence leaps out to me that the thing that has constituted the very essence and power of this work, has been the face which, in today's world, is the most neglected, when it is not frankly treated as an object of derision or disdainful condescension: that of the ideas, even that of dreams, never that of results.

In attempting in these pages to discern the most essential aspects of my contribution to the mathematics of our time, via a comprehensive vision that choses the forest over the trees - I've observed, not a victorious collection of "grand theorems", but rather a living spectrum of fertile ideas, which in their confluence have contributed to the same immense vision.(*)

(*)That does not my work is lacking in major theorems, including those theorems which resolve questions posed by others , which no-one before myself had known how to solve. (Some of these are reviewed in the note at the bottom of the page (***) pg. 554, or the note " The rising sea..." (R&S, #122.) Yet, as I've already emphasized right at the beginning of this "promenade" (#6 "Vision and points of view), these theorems assume meaning for me only within the nourishing context of a grand theme initiated by one of those "fertile ideas". Their demonstration follows from them, as from a spring and effortlessly, even from their very nature, out of the "depths" of the theme that carries them - like the waves of a river appear to emerge calmly from the very depths of its waters, without effort or rupture. I've expressed the same idea, though with different images, in the footnote cited above , " The rising sea....".

The Child and its Mother

When, in the course of writing this "preface", I began this promenade through my work as a mathematician, (with its brief sketches of "inheritors" (authentic) , and "builders" (incorrigible)) , a name suggested itself by which this incomplete preface could be suitably designated . Originally it was " The child and the builders". Over the course of several days however, it became apparent that "the child" , and "the builder" were one and the same person. This appelation thereby became , simply, "the child builder"; a name, indeed, not lacking in charm, with which I was well pleased.

Yet it was revealed further along in the course of this reflection that this haughty "builder" or, (with more modesty) , The child who plays at making houses was nothing more than one of the two avatars of the child-who-plays. There is, in addition, the child-who-loves-to-investigate-all-things , who delights in digging in and being buried by the sands, or in the muddy sludge, all those exotic, impossible surroundings ... To indicate this change (if only for myself), I started to speak of him by means of the flashy word, the " pioneer"; followed by another more down to earth, though not lacking in prestige, the "explorer". I was then led to ask which, between the "builder" and the "pioneer-explorer", is the more masculine, the more enticing of the two? Heads or tails?

Following which, scrutinizing ever more closely, I beheld our intrepid "pioneer" who finds himself ultimately become a girl (whom I would have liked to dress up as a boy) - sister to pools, the rain, the fogs and the night, mute and virtually invisible from the necessity of staying always in the shadows - she whom one always forgets (when one is not inclined to mock her) ... And I as well found opportunities as well, for days at a time, to forget her - to do so doubly, one might say: I tried to avoid seeing anything but the boy (he who plays at making homes)- and even when it became impossible all the same to deny the other, I still saw her somehow in the guise of a boy ...

As a suitable name for my "promenade" in fact, it doesn't work at all. It's a phrase which is totally "yang", totally "macho", and it's lame. Not to appear biased it would have to also include the other But, strange as it may seem , the "other" really doesn't have a name. The closest surrogate would be "the explorer", but that too is a boy's name, and there's no hope for it. The language itself has been prostituted, it lays traps for us without our being aware of it, it goes hand in glove with our most ancient prejudices.

Perhaps one could make do with "the child-who-builds and the child-who-explores". Without stating that one is a "boy", the other a "girl", that it's a kind of single boy/girl who explores while building and while exploring builds .... Yet just yesterday, in addition to the double-sided yin-yang that both contemplates and explores, another aspect of the whole situation emerged.

The Universe, the World, let alone the Cosmos, are basically very strange and distant entities. They don't really concern us. It is not towards them that the deepest part of ourselves is drawn. What attracts us is an immediate and tangible Incarnation of them, that which is close, "physical", imbued with profound resonances and rich in mystery- that which is conflated with the origins of our being in the flesh, and of our species - and of That which at all times awaits us, silently and ever welcoming, "at the end of the road". It is She, the Mother, She who gives us birth as she gives birth to the World, She who subdues the urges or opens the floodgates of desire, carrying us to our encounter with Her, thrusting us forwards towards Her, to a ceaseless return and immersion in Her.

Thus, digressing from the road on this unanticipated "promenade", I found, quite by accident, a parable with which I was familiar, which I'd almost forgotten - the parable of The Child and the Mother. One might look upon it as a parable of "Life in Search of Itself" . Or, at the simple level of personal existence, a parable of "Being, in its quests for things " .

It's a parable, and it's also the expression of an ancestral experience, deeply implanted in the psyche - the most powerful of the original symbols that give nourishment to the deepest levels of creativity. I believe I recognize in it , as expressed in the timeless language of archetypal images, the very breath of the creative power in man, animating flesh and spirit, from their most humble and most ephemerable manifestations to those which are most startling and indestructible.

This "breath", even like the carnal image that incarnates it, is the most unassuming of all things in existence. It is also that which is most fragile, the most neglected and the most despised ...

And the history of the vicissitudes of this breath over the course of its existence is nothing other than your adventure, the "adventure of knowledge" in your life. The wordless parable that gives it expression is that of the child and the mother.

You are the child, issued from the Mother, sheltered in Her, nourished by her power. And the child rushes towards the Mother, the Ever-Close, the Well-Understood - towards the encounter with Her, the Unlimited, yet forever Unknowable and full of mystery ...

This ends the "Promenade through the life's work of a mathematician"

THE QUEST FOR ALEXANDRE GROTHENDIECK

Although his productive research ended in 1975, many mathematicians maintain that Alexandre Grothendieck is the greatest living mathematician. He has a well-earned reputation for political engagement and extreme eccentricity. In 1975, at the end of a 15 year period of astonishing mathematical creativity , he withdrew from active scientific life . Eventually he dropped out of all professional activities in Paris and took up an ordinary post in Montpellier as a teacher of elementary subjects such as linear algebra and calculus. Most of his energy was invested in radical political causes, anti-militarism and organic farming.

In May of 1988 the Swedish Academy awarded him the Crafoord Prize. This prize, a belated attempt to repair the neglect of Alfred Nobel in not creating a prize in mathematics, came with a cash award of about $160,000. Grothendieck stunned the world by rejecting it. In his letter to Le Monde he speaks of dishonesty in science and a political establishment that he refused to endorse by accepting the prize. He also hints at mystical visions forecasting an inexorable Holocaust, to be followed by a Utopian Golden Age.

I was in France at that time. After reading his letter in Le Monde I set out to find him. The adventurous search took 3 months. We took a liking to each other. During our interview and in our correspondence over the next few years, he shared ideas on the arms race, the purposes of science, the reasons for his rejection of the prize, plagiarism and dishonesty in mathematics and the scientific community in general.From him, and from local friends, I learned details of his biography that were not generally known.

The story of my search , our conversations and their sequel, was serialized in my newsletter Ferment in 1989 .

In 1992 he went into hiding , severing contacts with friends, family and colleagues. The author organized a committee to search for him that led to his discovery, in good health and busily at work, in September, 1996. This committee has since become the Grothendieck Biography Project. Contact me for information about our activities.

All of this is recorded in a 180 page account in 3 parts.

Harvests and Sowings

Thematic Inventory, or

Prelude in 4 Movements

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